

# JOURNAL OF TELECOMMUNICATIONS AND INFORMATION TECHNOLOGY

3 / 2020

## Differentiated Service Model-Supported Cluster-Based Routing in Wireless Sensor Networks

*J. Agarkhed, P. Y. Dattatraya, and S. Patil*

*Paper*

1

## Modeling and Analysis of Additional Clear Channel Assessment to Improve Performance of IEEE 802.15.4-based MAC Protocol

*M. G. Asuti and P. I. Basarkod*

*Paper*

10

## EARPC – Energy Aware Routing Protocol for Cooperative MIMO Scheme in WSNs

*K. C. Reddy, G. D. Devanagavi, and Thippeswamy M. N.*

*Paper*

22

## ICBCA – Improved Cluster Based Channel Allocation in Cognitive Radio Sensor Networks

*J. Agarkhed and V. Gatate*

*Paper*

28

## Cooperative Spectrum Sensing in Cognitive Radio Networks: A Survey on Machine Learning-based Methods

*S. Khamayseh and A. Halawani*

*Paper*

36

## Performance Improvement for Vehicular Communications Using Alamouti Scheme with High Mobility

*M. A. Youssefi and A. Mouhsen*

*Paper*

47

## Fuzzy-based Description of Computational Complexity of Central Nervous Systems

*P. Prokopowicz, D. Mikolajewski, K. Tyburek, and P. Kotlarz*

*Paper*

57

## Application of Graph Theory Algorithms in Non-disjoint Functional Decomposition of Specific Boolean Functions

*T. Mazurkiewicz*

*Paper*

67

(Contents Continued on Back Cover)

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ISSN 1509-4553      on-line: ISSN 1899-8852

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**Ministry of Science  
and Higher Education**  
Republic of Poland

Improvement of the linguistic quality of the journal; Assignment of DOI numbers; Access to the antiplagiarism system; Editing and printing of the journal – tasks financed under 706/P-DUN/2019 agreement from budget of the Ministry of Science and Higher Education under the science dissemination fund.



# Differentiated Service Model-Supported Cluster-Based Routing in Wireless Sensor Networks

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<https://doi.org/10.26636/jtit.2020.139119>

**Abstract**—Wireless Sensor Network finds its extensive use in healthcare applications for the transfer of time-critical data through wireless connectivity. The primary cause of network failure is the transfer of time-critical multimedia data. The article presents a new differentiated service model-supported (DSM) cluster-based routing in wireless sensor networks (WSNs) that overcomes the above issue. DSM prioritizes the transfer of different flow types based on packet type and packet size. The employment of computational offloading minimizes delay for critical and small-sized data packets and by carrying out data reduction of large-sized packets at proxy server. It outperforms the existing protocols in terms of energy efficiency, throughput, and reliability by prioritizing the transfer of time-critical health application data.

**Keywords**—clustering, energy efficiency, proxy server, TDMA, wireless sensor networks.

## 1. Introduction

A Wireless Sensor Network (WSN) consists of a group of randomly deployed sensor nodes that are capable of detecting various environmental phenomena, such as air pressure, temperature, humidity, etc. It sends the data collected to the base station referred to as the sink [1]. Different topologies are used as far as connectivity between the nodes is concerned, e.g. star, mesh and tree-shaped. Among the three, tree-based routing is the best solution for node communication. A mesh network incurs a routing overhead and requires the provision of a routing table. Compared to the star-based topology, tree-based routing protocols are characterized by lower energy consumption, as they rely on multi-hop communication.

Several techniques exist for minimizing energy consumption and increasing network lifetime [2], [3] by relying on energy-harvesting [4]–[9]. Power consumption is minimized by duty cycling [10]–[12] and data reduction techniques. In this work, a differentiated service model-supported cluster-based routing in WSN (DSM) is presented [12]. It provides a reliable and energy efficient method for transferring data from a sensor node to the

cluster head (CH), using an adaptable path that is based on packet data type. DSM prioritizes the forwarding of time-critical data along one primary routing path through queue 1. A reduction in data quantity at the sensor node is based on computation time. Small size packets are used to reduce the amount of data at the sensor node, and are passed through queue 2, while the large size packets are passed through queue 3 and are subjected to data reduction techniques later, at the proxy server. The packets compressed at the sensor node are passed through queue 2. The uncompressed packets are passed through queue 3, and emergency packets are passed through a queue 1 to support service differentiation.

Further, these packets are transmitted simultaneously, which results in the bandwidth of all paths being aggregated to achieve high throughput and reduced delay. By the use of different traffic flows for different service classes, interference between different packet types is avoided. A Local Energy Consumption Prediction-based clustering protocol for Wireless Sensor Networks (LECP) [14] lack this feature and packets flowing through the same path are subject to high interference and delay leading to retransmission of lost packets. The DSM is an extension of Energy efficient service differentiated QoS aware routing in cluster-based wireless sensor network [13] with simulation graph obtained by a varying number of nodes, interval, and simulation time. The remaining sections of this paper are organized as follows. Section 2 presents details concerning the related work, and Section 3 presents the proposed technique. Section 4 lists the simulation parameters with performance evaluation, and Section 5 concludes the work.

## 2. Related Work

In [15], the author points out that WSN nodes rely on a limited capacity of the power source, restricted communication bandwidth, processing speed and memory space. These limitations have given rise to numerous research projects and studies which focused exclusively on maximizing the utilization of the limited sensor resources [16].

Efficient use of power during data transmission is achieved by adopting an approach known as clustering. Clustering helps improve network scalability, load balancing, and allows to reduce the size of the routing table at individual nodes [17], [18] in order to save the communication bandwidth by relying on inter-cluster interaction among CHs [19]. It is also useful for providing a stabilized network topology [20]. To conserve energy, CH may schedule cluster activities by switching a given node to active and sleep modes [21], [22]. The following is another advantage of clustering: CH aggregate data from all sensors in their respective clusters to reduce the count of packets to be sent [23]. In [24], the author proposed non real-time and real-time traffic transmission solutions based on the initial energy of sensor nodes derived from different energy levels, inverse expected transmission count, expected transmission count, and minimum loss. In papers [25], [26], an incremental approach to a given session, in which high bandwidth is required to satisfy the needs of multimedia applications, is proposed by relying on non-interfering paths in order to support an increased lifetime of each node. In [26], the author discussed a routing protocol related to healthcare and the need of using the multipath approach in order to achieve an optimal solution to satisfy the QoS requirement of the critical application. Based on the study of related work, one may identify one crucial aspect that need to be met: QoS parameters, such as data accuracy, minimized delay and minimized energy consumption, need to be optimized in order to improve the lifetime of a sensor node.

### 3. Proposed Technique

The proposed technique begins with cluster formation and the complete operation specified in terms of rounds. The clustered topology formed by dividing the cluster setup phase into sub-phases is presented in Fig. 1. The first sub-phase consists in network initialization and in predicting energy consumption. The CH selection sub-phase occurs next and, finally, cluster formation takes place, followed by route construction.

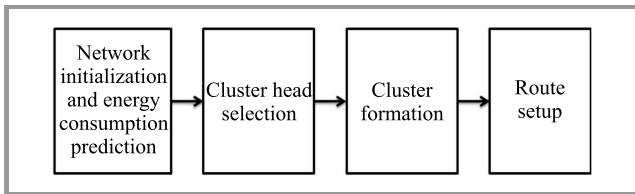


Fig. 1. Cluster formation and route setup.

Each node first broadcasts a Node\_Msg containing the ID and current energy within radius  $Ra$ . The node also accepts the Node\_Msgs sent by its neighbors. Upon receiving this information, each node computes the distance to each of its neighbors based on the strength of the received signal, i.e. Received Signal Strength Indication (RSSI). Then

RSSI value is used to compute the energy consumption ratio  $E_{ratio}$  used to select the CH. For any node  $N_i$ , its  $E_{ratio}$  computed in time interval  $T_1$  is:

$$E_{ratio}(N_i) = \sum_{i=1}^n \frac{E_{con}(N_i)}{E_{cur}(N_i)}, \quad (1)$$

where  $E_{con}$  – energy consumed by node  $i$ ,  $E_{cur}$  – current energy of node  $i$ ,  $n$  – number of sensor nodes within the range of node  $N_i$ .

The node with the lowest energy consumption ratio is selected as CH.

CH selection commences after time interval  $T_1$ . Each node compares its energy consumption ratio with that of other nodes and the node with the lowest  $E_{ratio}$  is selected as being capable of assuming the role of CH. If the node does not receive the Head\_Msg after time interval  $T_2$ , then it broadcasts the Head\_Msg within radius  $Ra$  to announce itself as CH and prepares to receive the Join\_Msg to create a TDMA schedule for all CM within range  $Ra$ . If the node receives the Head\_Msg, then it sends the Join\_Msg and waits to receive a TDMA schedule.

If a node is not selected as CH, then it finds the nearest CH and sends the Join\_Msg to that CH. The Join\_Msg contains the ID of the node and its current energy. Then, in each cluster, CH creates a TDMA schedule based on the Join\_Msgs received and broadcasts the TDMA schedule to all CMs. After the end of this round, the data transmission phase occurs.

#### 3.1. Route Setup

The proposed cluster-based approach employs two types of communication: direct and intra-cluster communication.

In intra-cluster routing, every CM node collects data periodically from the environment and checks the data stream type. Packets with a small data size are compressed and sent to CH. Then, CH classifies the data packet as an emergency data packet, a compressed data packet or an uncompressed data packet. The collected data are sent, within a given time slot, to the CH based on the TDMA schedule. The transmission of data within a given time slot allows to avoid packet collision and retransmission in the same cluster. Upon collecting data from CMs, the CH aggregates the data and performs data transmission based on a routing tree, along a path that satisfies inter-cluster routing.

For inter-cluster communication, a routing tree is created between the CHs within time  $T$ . Each CH broadcasts a route request. CH  $i$  chooses its next-hop node as a base station if its distance to the base station is lower than Euclidean distance. Else, it chooses node  $j$ , having a minimum distance to the base station and a minimum  $E_{ratio}$  among all its neighboring CH nodes (Figs. 2 to 5).

As depicted in Fig. 2, the N2 is the source node, while N1, N3, and N5 are chosen as candidate nodes for the next-hop based on their distance to the nearest BS. The subsequent node with the lowest energy consumption ratio is chosen as a next-hop. Finally, N5 has the lowest  $E_{ratio}$  among N1,

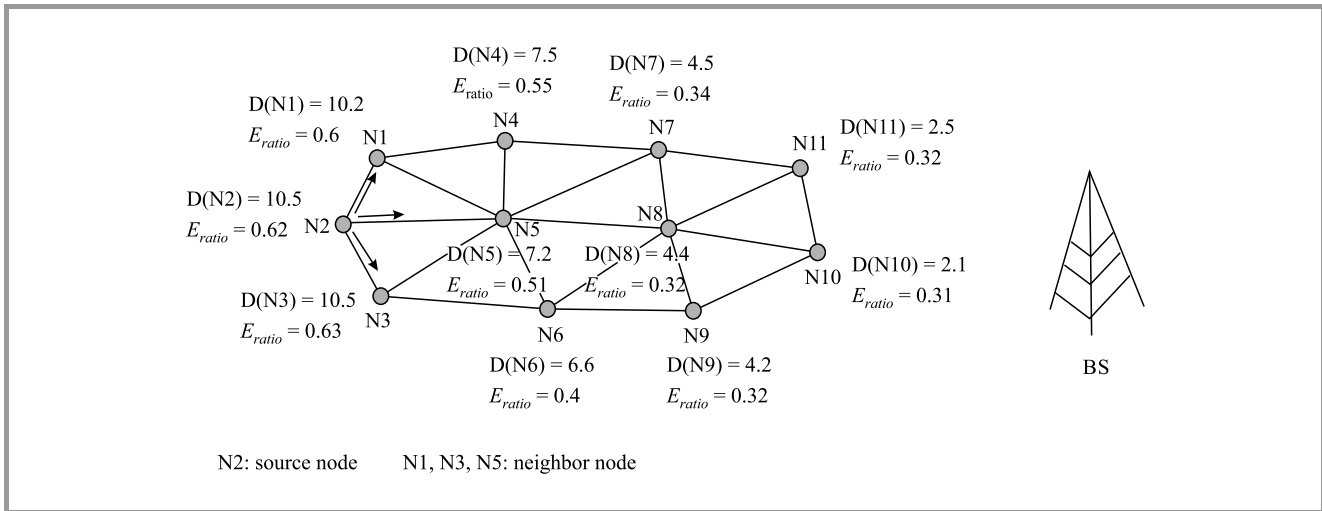


Fig. 2. Choosing candidate nodes from the N2 node.

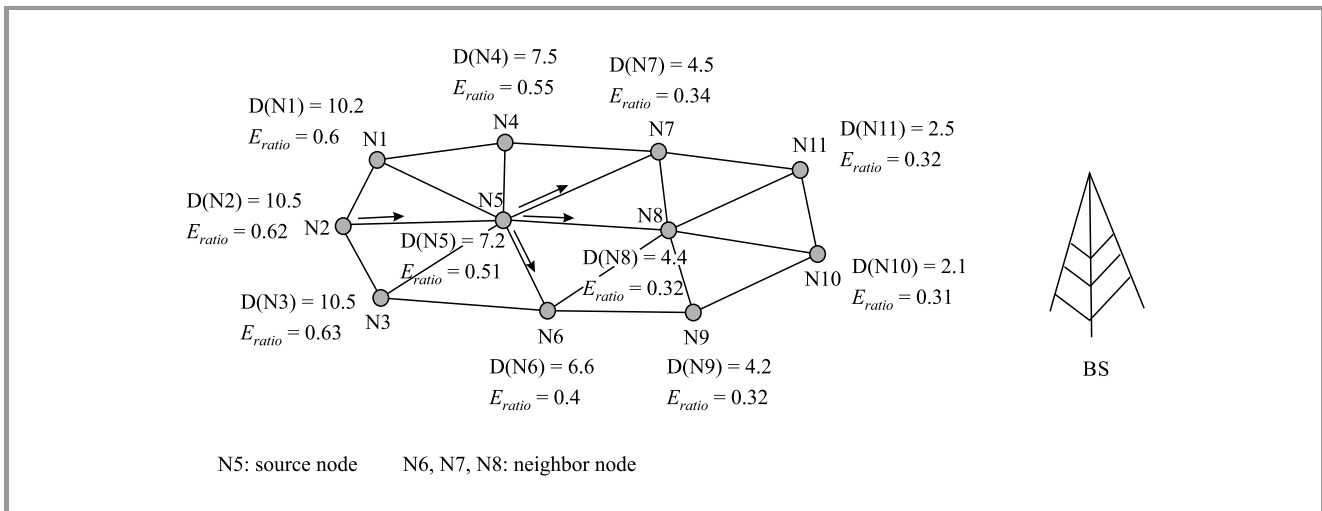


Fig. 3. Choosing candidate nodes from the N5 node.

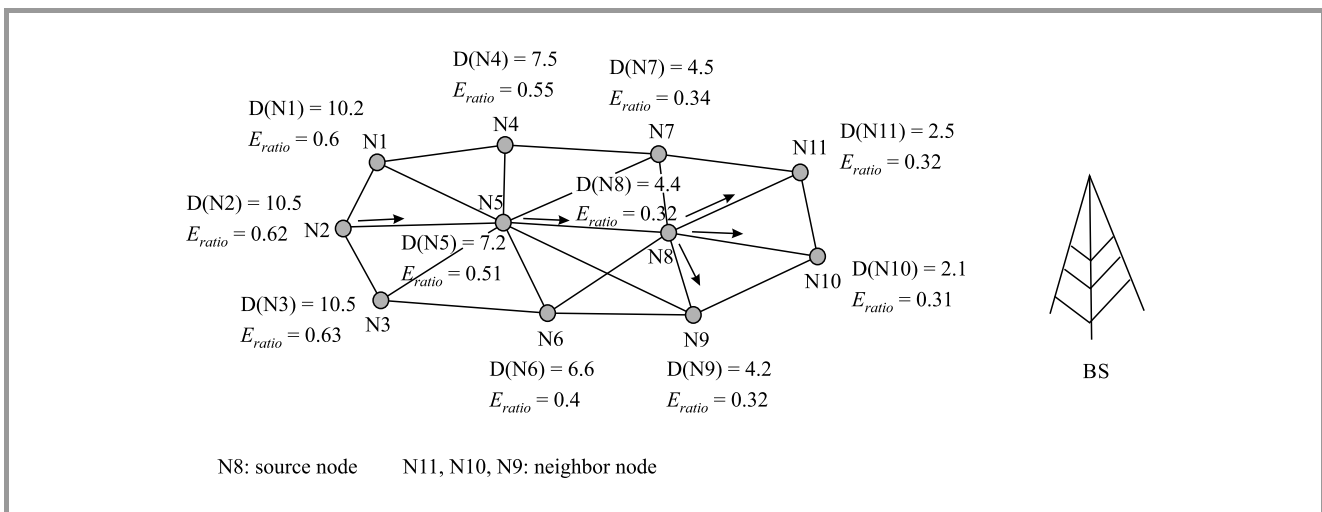


Fig. 4. Choosing candidate nodes from the N8 node to BS.

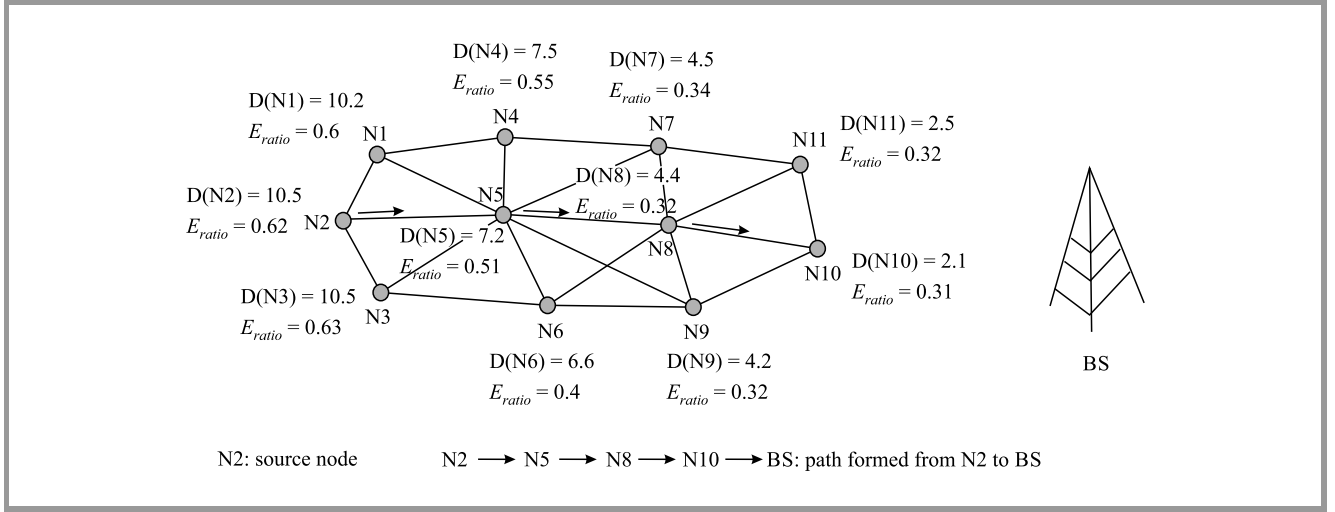


Fig. 5. Path established from node N2 to BS.

N3, and N5 nodes and is selected as the next-hop within the tree.

As shown in Fig. 3, N5 is the source node, while nodes N6, N7, and N8 are chosen as candidate nodes for the next-hop based on their distance to the nearest BS. The subsequent node with the lowest energy consumption ratio  $E_{ratio}$  is selected as the next-hop. The N8 has the lowest  $E_{ratio}$  value among N6, N7, and N8 nodes and is selected as the next-hop within the tree.

As depicted in Fig. 4, the N8 is the source node, while nodes N9, N10, and N11 are chosen as candidate nodes for the next-hop based on their distance to the nearest BS. The subsequent node with the lowest energy consumption ratio is selected as the next-hop. Node N10 has the lowest value among the N9, N10, and N11 and is chosen as the next-hop node. Finally, the path is constructed from source node N2 to the sink through intermediate N5, N8, N10 nodes, as depicted (Fig. 5).

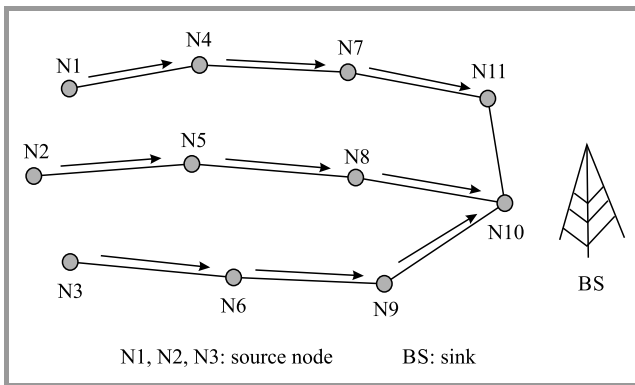


Fig. 6. Path constructions from node N1, N2, and N3 to BS.

Figure 5 shows the path from source node N2 to the sink node, through intermediate nodes N5, N8, N10, while Fig. 6 shows a routing path from nodes N1, N2, and N3 to BS.

#### Algorithm 1: DSM

**Input:** Packet with the different data stream

**Output:** Prioritized packets directed on different paths

1. Deploy heterogeneous sensor nodes
2. Base station broadcast hello packets to all nodes
3. Node local energy consumption calculation
  - 3.1. Calculate nodes present energy:
 
$$E_{pr} = E_{ini} - E_{tr}, \quad (2)$$
 where  $E_{pr}$  – node present energy,  $E_{ini}$  – node initial energy,  $E_{tr}$  – energy consumption for data transmission
- 3.2. Update nodes information
  - 3.2.1. Store BS location
  - 3.2.2. Acknowledge BS with node location, node type, and neighbor list
4. Select CH as a node with maximum present energy and remaining nodes as cluster members
5. Schedule nodes within the cluster using TDMA
6. Calculate energy consumed at the sensor for data reduction

$$E_{sn} = P_{sn} \cdot \frac{T_c}{S_{sn}}, \quad (3)$$

where  $E_{sn}$  – energy consumed at sensor node,  $P_{sn}$  – power consumed by sensor node,  $T_c$  – total computation,  $S_{sn}$  – speed of sensor

7. Calculate power consumed at proxy server for data reduction

$$E_{ps} = P_{td} \cdot \frac{D_t}{B} + W_p \cdot \frac{T_c}{S_{ps}}, \quad (4)$$

where  $E_{ps}$  – power consumed at the proxy server,  $\frac{D_t}{B}$  – energy consumed to transmit data  $D_t$ , to proxy in given bandwidth  $B$ ,  $P_{td}$  – data transmission power,  $W_p$  – power spent by server waiting for data,  $S_{ps}$  – proxy server speed

8. **If**  $E_{sn} < E_{ps}$  **then** perform data reduction at sensor node and send packet on a different path  
**Else** perform data reduction at the proxy server and send the packet on a different path
9. Data transmission from the sensor node to CH and from CH to BS
10. Performance evaluation
11. End

### 3.2. DSM Algorithm

The proposed protocol relies on clustering to reduce transmission energy, using the inter-cluster communication routing tree construction algorithm, based on the local energy consumption ratio of nodes [26], and also provides differentiated service as illustrated in Algorithm 1.

Energy consumption, delay, and overall routing overheads have been minimized and throughput has been increased by DSM, as shown in simulation graphs, due to its efficient cluster setup and routing tree construction, as depicted in Figs. 7 and 8.

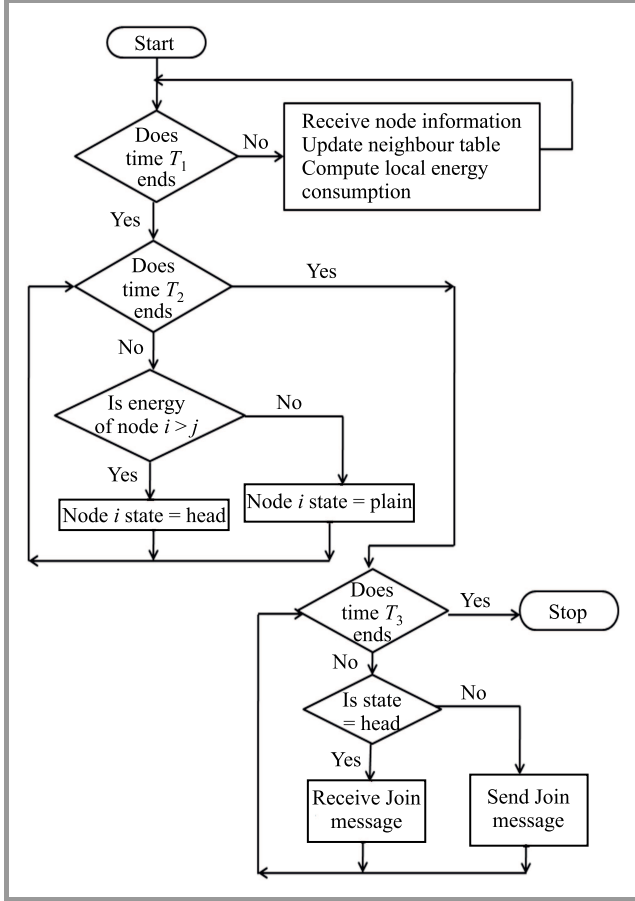


Fig. 7. Cluster setup based on local energy consumption ratio.

In the cluster formation phase, each node broadcasts a message with node ID and current remaining energy. After receiving the RSSI value from its neighbor node, each node computes its local energy consumption ratio. All nodes complete this process in time  $T_1$ . If a node has a local energy consumption ratio that is lower than all other results, then it has a good chance of becoming a CH. At the end of time  $T_2$ , if the node does not receive the HeadMsg and has more energy than other nodes, then it announces itself as CH. In time  $T_3$ , if the state of the node is CH, then the node receives the Join message from a neighbor node. If a state of the node is Plain, then it sends the Join message to CH and becomes a CM.

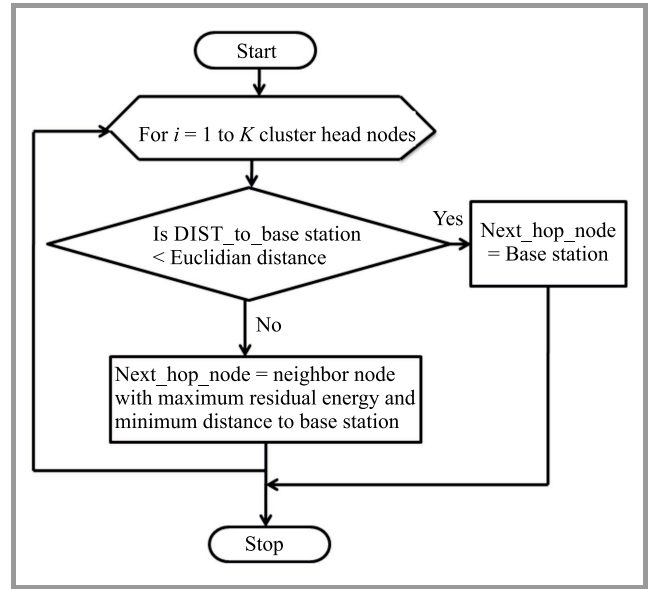


Fig. 8. Routing tree construction.

For inter-cluster communication, routing tree is constructed among CHs in the network, as depicted in Fig. 8. Each CH broadcasts the RouteMsg and checks whether the neighbor node has the highest residual energy and that its distance to BS is the lowest among the set of neighbor nodes. Next, CH  $i$  chooses its next-hop node as BS if its distance to BS is lower than Euclidean. Else, it selects node CH  $j$  that has a minimum distance to the base station and the lowest local energy consumption ratio among all its neighbor CH nodes.

## 4. Performance Evaluation

The performance of the proposed DSM technique has been evaluated using the NS2 simulator. The evaluation setup includes a random deployment of 120 nodes within an area measuring  $1000 \text{ m} \times 500 \text{ m}$  area. The base station is positioned 50 m away from the deployment area, as shown in [26]. Figure 9 shows the simulation scenario consisting of 100 nodes positioned throughout the sensor field and a proxy server that supports energy-efficient data collection using the DSM protocol. The channel capacity is set to  $3 \cdot 10^6$  for each node. IEEE 802.15.6 is used at the MAC layer. The simulated traffic is ECG-, pulse- and breath-generated at the application layer (Table 1).

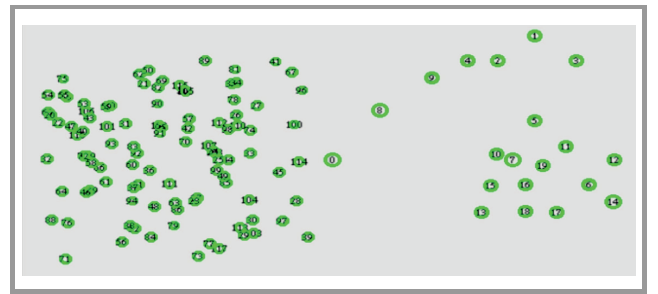


Fig. 9. Simulation scenario.

Table 1  
Simulation parameters list

| Parameter                            | Value |
|--------------------------------------|-------|
| Total number of nodes                | 120   |
| Number of clusters                   | 15    |
| Simulation time                      | 200 s |
| Traffic source                       | CBR   |
| Transmit power                       | 0.2 J |
| Receive power                        | 0.1 J |
| Initial energy                       | 100 J |
| Number of user nodes among 120 nodes | 10    |
| Storage device                       | 1     |
| Service provider                     | 6     |
| Task divider                         | 1     |
| Proxy server                         | 1     |
| Sensor nodes                         | 100   |

Performance of DSM was evaluated by varying packet size, simulation time and intervals. The results are obtained by varying packet size, interval and simulation time. Figure 10 shows that an increase in the data rate does not affect the packet delivery ratio, as the packet classifier forwards different packet types into various queues. Emergency and compressed packets in queue 1 and queue 2 are sent with a high priority and are not subjected to delays caused by large size packets in queue 3.

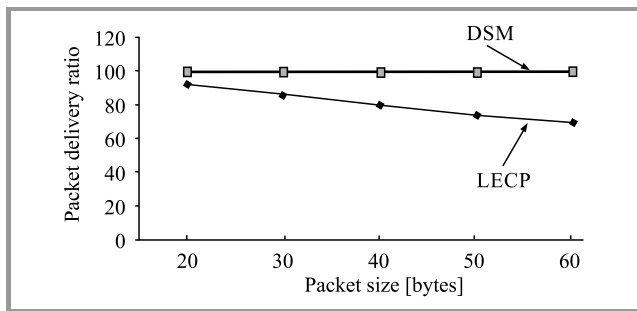


Fig. 10. Packet size versus packet delivery ratio.

As the data rate increases, DSM suffers from a minimal packet drop compared to LECP, thanks due to the separation of data packets into queues with different priority rates (Fig. 11).

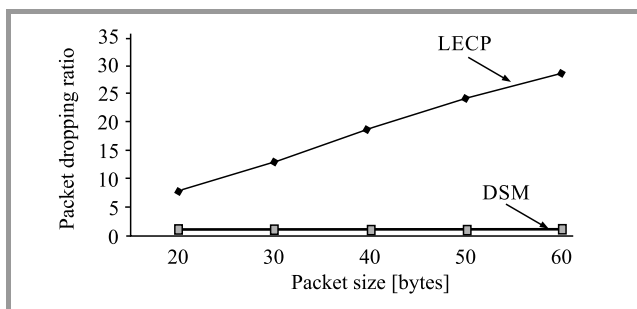


Fig. 11. Packet size versus packet dropping ratio.

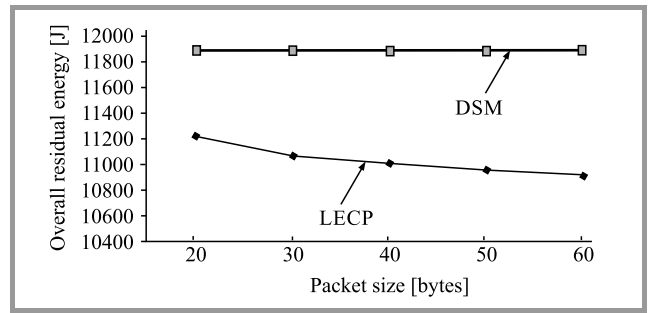


Fig. 12. Packet size versus overall residual energy.

Figure 12 shows that overall residual energy is minimized with an increase in packet size, as the sensor node performs a data size reduction if a shorter data reduction time is required. If the data reduction time at the sensor is high, then the data are sent to the proxy server for compression. Avoiding data reduction at the sensor node limits energy consumption. LECP needs 65% more energy compared to DSM.

The DSM method achieves a normalized routing overhead by transmitting data from each sensor node to CH in a cluster-based form, using a stochastic schedule [26] (Fig. 13).

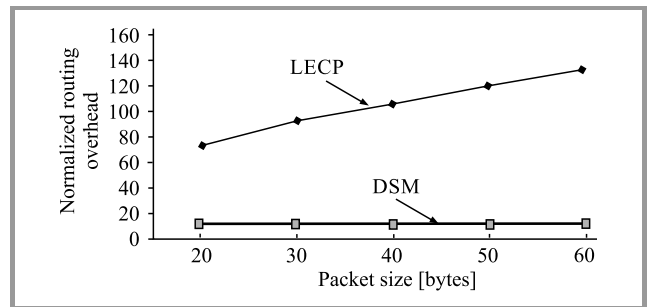


Fig. 13. Packet size versus normalized routing overhead.

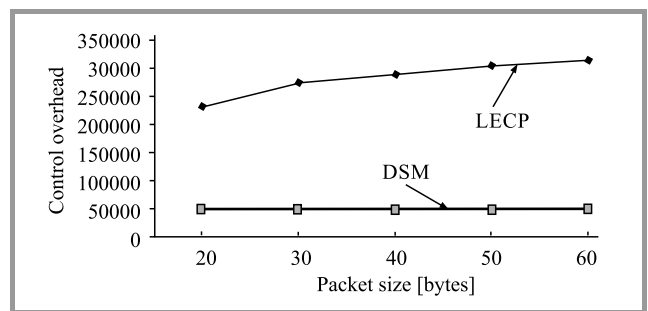


Fig. 14. Packet size versus control overhead.

Figure 14 shows that the DSM method has a decreased routing overhead compared to LECP, as there is no control message transfer overhead among the nodes. DSM balances the traffic load by separating packets into different service types and avoids packet drops by minimal interference between packets of same type. This, in turn, avoids retransmission and transfer of control messages.

The DSM method shows less jitter compared to LECP thanks to the distribution of packets via different flows (Fig. 15). The packets in each queue are subjected to a delay caused by the same type of packets, which may be lower and which generates minimized jitter with priority offered to emergency data packets.

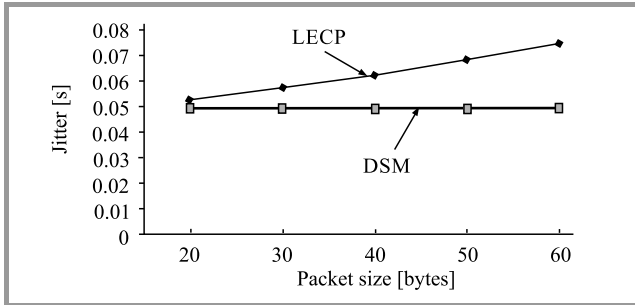


Fig. 15. Packet size versus jitter.

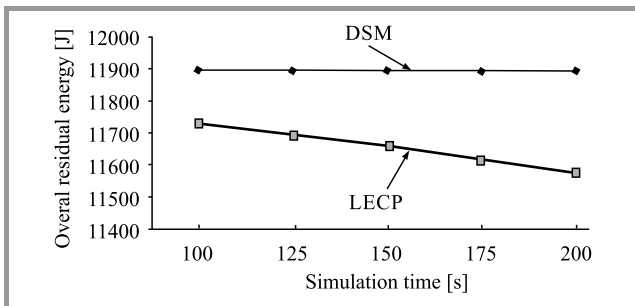


Fig. 16. Overall residual energy.

Figure 16 shows that the overall residual energy remains constant, with an increase in simulation time, as different traffic flows are used for different service classes. Such an approach avoids interferences between different packet types. LECP lacks this feature and packets flowing through the same path are subjected to more interference and delay, consequently leading to retransmissions.

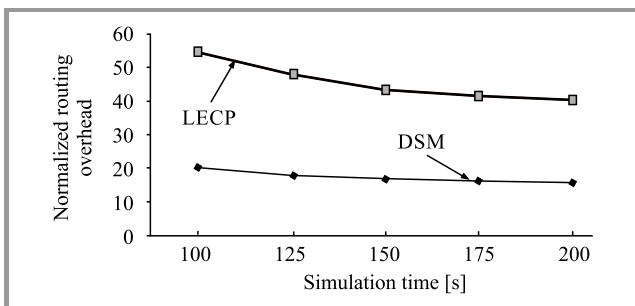


Fig. 17. Normalized routing overhead.

The DSM has a 25% lower normalized routing overhead due to the use of cluster-level node scheduling based on sensing error (Fig. 17). Also, DSM incurs a minimal delay compared to LECP, by using service differentiation and by prioritizing emergency data packets (Fig. 18). In

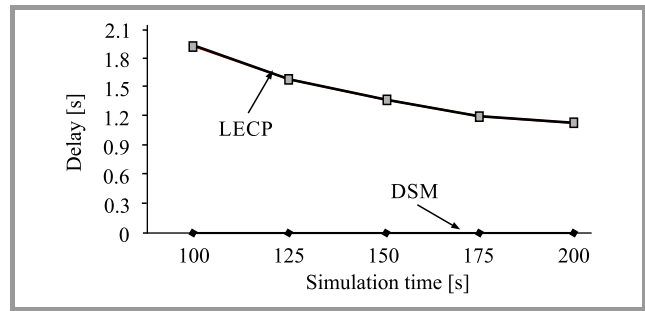


Fig. 18. Delay.

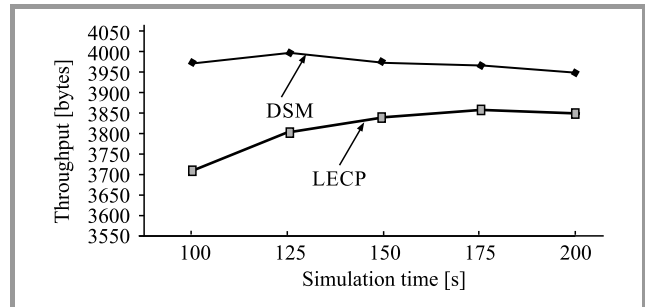


Fig. 19. Throughput comparison.

the case of LECP, large size packets may increase the delay for subsequent small size packets. Large size packets and small size packets are separated into different flows in DSM. The throughput is higher in DSM than in LECP thanks to the use of computation offloading (Fig. 19).

Next, simulations in the interval domain were performed. Here, the DSM method displays a constant and lower delay with the increasing number of packets (Fig. 20). At interval 6, with the maximum number packets, DSM shows a slight increase in delay that is lower, however, than 0.005 s, which is negligible compared to LECP that has a delay of 6.18 s for the same number of packets.

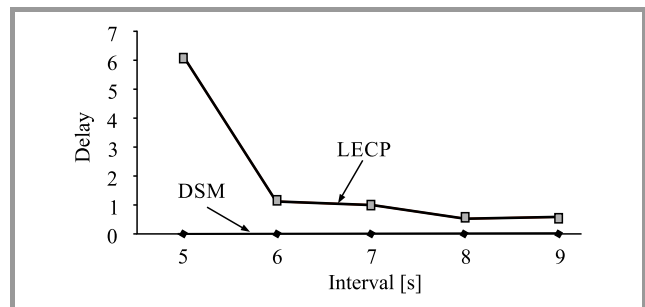


Fig. 20. Interval [s] vs. delay [s].

Figure 21 shows that throughput achieved using DSM is higher when compared to LECP, thanks to the scheduling of nodes and cluster-based routing. As the interval increases, the number of packets transmitted per second decreased, which – in turn – may lead to a less collision of packets. At the interval of 5 s, DSM achieves higher throughput compared to LECP for the transmission of a minimal number of packets.



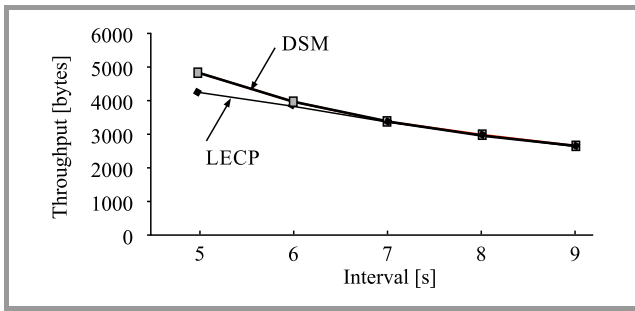


Fig. 21. Interval vs. throughput.

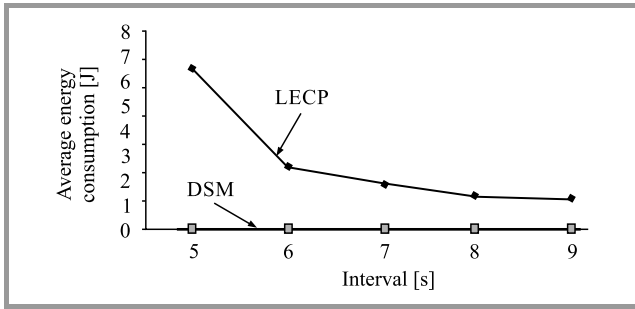


Fig. 22. Interval vs. average energy consumption.

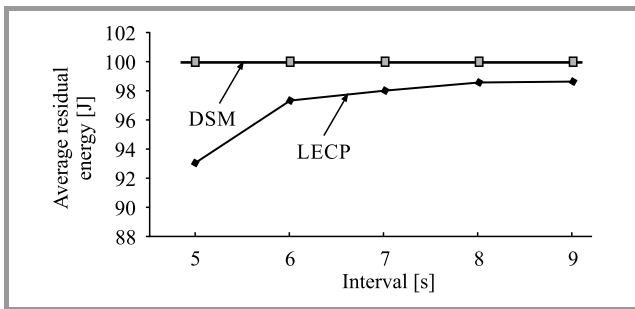


Fig. 23. Interval vs. average residual energy.

The average energy consumption observed in DSM is minimal compared to LECP, even with an increase in the number of packets, thanks to the use of offload computation as depicted in Fig. 22. Also, DSM achieves a higher average residual energy compared to LECP, with an increase in the number of packets per seconds as depicted in Fig. 23.

## 5. Conclusion

Avoiding delay is one of the most significant issues in critical data applications. The DSM method transfers time-critical data by assigning them with high priority and relies on service differentiation to separate data into emergency and non-emergency flows. Emergency data is sent with a top priority, while non-emergency data is compressed at the sensor node to minimize transmission energy consumption. If energy consumed at the sensor node is less than at the proxy server, then data reduction is carried out at the sensor node. If the data compression

at the sensor node consumes more energy than at the proxy server, then the data reduction task is offloaded to the proxy server in order to minimize the delay for emergency and small size packets. DSM helps differentiate between packets, providing higher service quality during the flow of data over the prioritized path, utilizing network bandwidth efficiently with a minimized delay and routing overheads.

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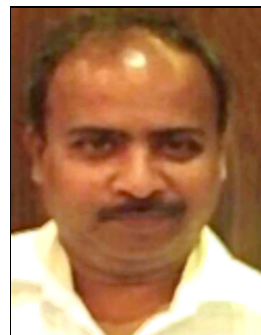
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# Modeling and Analysis of Additional Clear Channel Assessment to Improve Performance of IEEE 802.15.4-based MAC Protocol

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<https://doi.org/10.26636/jtit.2020.140420>

**Abstract**—Design of the MAC protocol is crucial in all wireless sensor networks (WSNs) due to its influence on the performance of the transceiver, i.e. the most energy-consuming component of each sensor node. A mechanism known as “carrier sense multiple access with collision avoidance” (CSMA/CA) is used for accessing the wireless channel in the IEEE 802.15.4 standard-based MAC protocol in order to avoid collisions between the network’s communicating nodes. CSMA/CA relies on two clear channel assessments (CCA=2) for checking the status of the channel. In this paper, we develop an additional CCA algorithm for the two scenarios encountered in star topology-enabled WSNs. Next, we investigate the impact of an additional clear channel assessment (CCA=3) on performance in IEEE 802.15.4. We develop a Markov chain model for the proposed methodology, and validate it using Matlab. Simulation results show that there is a significant improvement of performance metrics in the IEEE 802.15.4 standard-based MAC protocol with an additional CCA.

**Keywords**—clear channel assessment, CSMA/CA mechanism, IEEE 802.15.4, MAC protocol, wireless sensor networks.

## 1. Introduction

Short range, low rate communication systems, such as low rate wireless personal area networks (LR-WPANs) find various applications in smart meters, industrial sensing, health care, home automation and monitoring. The IEEE standard-based definition used for low rate transmissions describes the physical and MAC layers of wireless sensor networks. The MAC layer that complies with the IEEE 802.15.4 standard [1]–[5] uses the “carrier sense multiple access with collision avoidance” (CSMA/CA) mechanism for accessing the channel. Current research focuses on the improvement of performance parameters of IEEE 802.15.4-based MAC protocols, in order to achieve different objectives, thus enabling numerous WSN applications. The standard CSMA/CA mechanism accesses the wireless channel (by checking its status), relying on clear channel assessment (CCA) with a standard value = 2. The status of the channel is checked twice to determine whether the medium is busy or idle, before commencing data transmis-

sion. Later on, based on the condition of the channel, the nodes will transmit data in the medium in order to avoid the probability of a collision within the network. Furthermore, CCA may be either of the single or double variety, depending on the periodic traffic pattern in the network [6].

IEEE 802.15.4-based WSNs may be based either on star or peer-to-peer topologies, depending on the application-related requirements. In star topologies, a personal area network (PAN) coordinator is necessary to establish communication between any pair of nodes.

In order to maintain better synchronization between the communicating nodes, the PAN coordinator adopts a superframe structure which specifies beacon interval (BI), active period, superframe duration and inactive period. The beacon signal is used for synchronization of the frames sent by the PAN coordinator to other nodes. BI constitutes the period estimated between the start and the end of a given frame. Therefore,  $BI = aBaseSuperframeDuration \times 2^{BO}$  symbols ( $BO \leq 14$ ). Active period is specified by the superframe duration parameter, given by  $SD = aBaseSuperframeDuration \times 2^{SO}$  symbols ( $0 \leq SO \leq BO \leq 14$ ). The BO attribute determines the beacon order and SO specifies superframe duration, i.e. the active part of the superframe structure.

The active period consists of 16 slots which are subdivided into contention access period (CAP) and contention free period (CFP). Under CAP, the nodes will utilize the slotted CSMA/CA CSMA/CA mechanism and the PAN coordina-

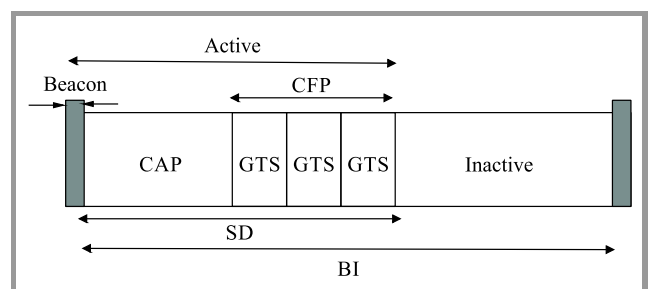


Fig. 1. Superframe structure of the IEEE 802.15.4 standard.

tor plays a major role during CFP in which the nodes send requests for guaranteed time slots (GTS) to the PAN coordinator. The inactive period provides details of the node in sleep state. Figure 1 illustrates the superframe structure utilized for the standard CSMA/CA mechanism.

### 1.1. CSMA/CA Standard

The CSMA/CA algorithm operates in two modes, namely beacon-enabled mode and non-beacon-enabled mode. Here, the beacon-enabled mode of the CSMA/CA mechanism is considered for channel assessment purposes. The standard initializes three parameters, i.e. number of backoff stages ( $NB = 0$ ), contention window ( $CW = 2$ ), and minimum value of backoff exponent ( $BE_{min}$ ). Backoff time is initialized for setting the delay in the range of  $2^{BE} - 1$ . The backoff period varies from  $BE_{min}$  to the max value of backoff exponent ( $BE_{max}$ ). CCA is performed for checking the status of the channel.

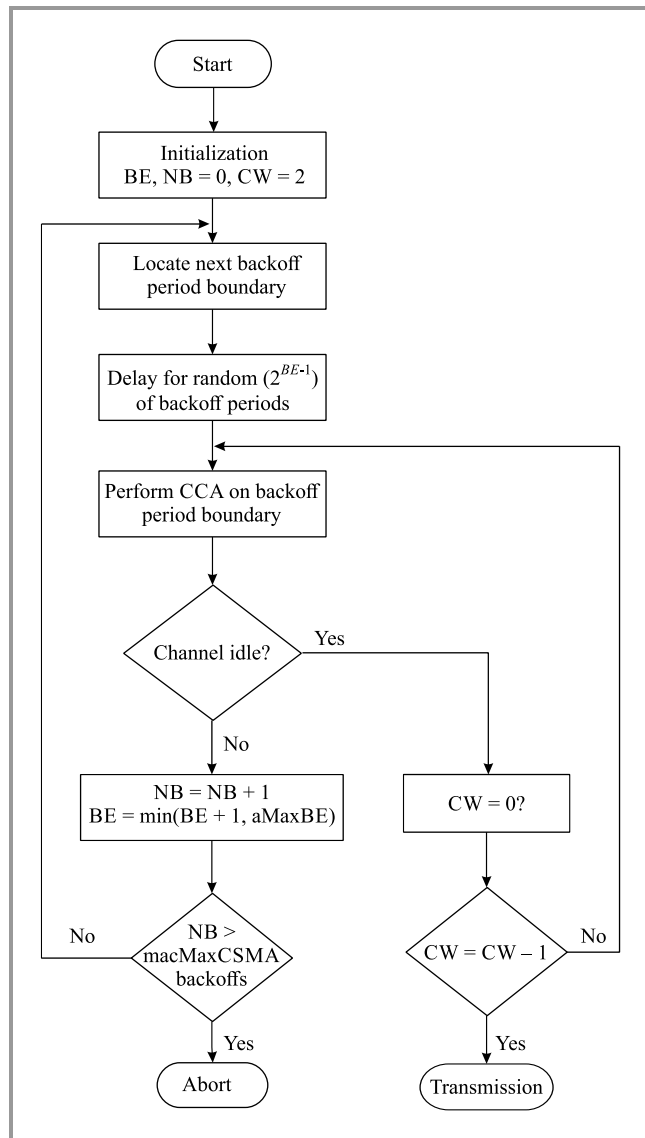


Fig. 2. CSMA/CA mechanism compliant with the IEEE 802.15.4 standard.

Upon identifying that the channel is busy, NB and BE are incremented by 1, and CW remains unchanged ( $CW = 2$ ). If the CSMA backoffs exceed the limit, the frames are dropped. Otherwise, the algorithm proceeds to the next. Upon determining that the channel status is free, CW is decremented by 1. When  $CW = 0$ , the data frame is transmitted by the node.

Figure 2 shows the CSMA/CA mechanism defined for the IEEE 802.15.4 standard. CSMA/CA adapts the number of CCAs (standard value = 2) to verify the status of the channel. However, in the existing CSMA/CA mechanism, some of the performance metrics are degraded for certain traffic scenarios. We need to address the issues and problems encountered in the CSMA/CA mechanism used in the IEEE 802.15.4 standard-based MAC protocol.

### 1.2. Research Contribution

In this paper, the impact of conducting an additional CCA covering some of the performance metrics of IEEE 802.15.4-based MAC protocol is investigated. The main contributions of this paper are summarized as follows:

- An accurate Markov chain model for the proposed additional CCA is developed to improve performance of IEEE 802.15.4-based MAC protocol. The proposed model is based on two scenarios encountered in the network due to two successive CCA (standard value = 2) failures.
- Based on the proposed model, the impact of the additional CCA on the performance metrics in the IEEE 802.15.4 standard is investigated.
- The performance of the additional CCA is evaluated based on extensive simulations performed using Matlab. Additionally, the proposed methodology is compared with the existing CCA approaches. Finally, the paper shows that by incorporating an additional CCA, it is possible to improve performance metrics of the IEEE 802.15.4 standard.

In Section 2, related work is presented concerning various CCA approaches introduced in the IEEE 802.15.4 standard. In Section 3, the problem is described for two scenarios considered in a star topology network. In Section 4, the proposed additional CCA algorithm is presented. In Section 5, the Markov chain model for the proposed additional CCA is shown. In Section 6, various performance metrics are developed and analyzed. In Section 7, performance evaluation is performed for the additional CCA and the results obtained are compared with the IEEE 802.15.4-based MAC protocol. Section 8 concludes the paper.

## 2. Related Work

In paper [7], the authors presented an additional carrier sensing algorithm for channel access mechanism in the IEEE 802.15.4 standard, improving overall throughput, power consumption, and MAC delay. In article [8], the authors studied the effect of CCA-related variation con-

duction on some of the performance metrics of the IEEE 802.15.4-based MAC protocol. Paper [9] proposed a new backoff scheme for the standard with an improvement in throughput and energy efficiency.

In [10], the authors adopted a new channel access mechanism referred to as the linearly increased backoff (LIB) method, with an enhancement of packet delay, throughput, and energy consumption. Authors of [11] developed a second chance CCA algorithm which consumes less energy and decreases the average delay required to conduct CCA. In [12], a channel access mechanism for the shared link existing between two or more devices in a time-slotted channel hopping (TSCH) network is presented. A model for performance metrics, such as average access delay, throughput, and packet loss rate is developed. In paper [13], the authors observed a variation in MAC parameters with fading channel conditions in IEEE 802.15.4. They argue that fading channel conditions positively affected power consumption and the number of packets received successfully in the network. The authors of [14] showed a new wireless channel access mechanism which improved reliability and reduced energy consumption of the node.

Article [15] shows a new approach to accessing channels, obtaining the precise status of a channel for packet transmission. The authors have proven that throughput increased by 8.76% and decreased the number of CCAs required by 3.9%. Paper [16] presents an efficient blind adaptive access parameter tuning algorithm which consumes less energy and improves the reliability of IEEE networks. The algorithm is based on reducing the extra overhead required for the backoff process without acquiring the acknowledgement signal. In [17], the authors developed a new channel access mechanism for IEEE 802.15.4 networks, providing channel access to all devices with a high buffer capacity, offering a minimum packet drop rate and high throughput. In article [18], the authors presented a new CSMA/CA mechanism based on different priorities assigned to the nodes, i.e. they came up with a classification of nodes based on three different priorities, such as fixed channel access for nodes with priority-0 and different probabilities for nodes with priority-1 and priority-2. The authors of [19] analyzed an increase in CSMA waiting time with specific MAC parameters being varied under different traffic conditions. They claimed that the collision probability was reduced when the  $\text{macminBE}$  value is increased. Article [20] shows a contention differentiated adaptive slot allocation CSMA/CA algorithm for wireless body area networks, reducing collision probability by minimizing packet retransmission.

In paper [21], a real-time scenario used in wireless body area networks for tuning some of the MAC parameters adaptively, in accordance with the IEEE 802.15.4 standard, as well as for maximizing the packet delivery ratio and reliability of the network is considered. In [22], a novel adaptive duty cycle algorithm for channel access mechanism in IEEE 802.15.4 networks, with adjustment of the duty cycle to different traffic conditions is proposed. It is proved that energy efficiency is improved for high traffic

conditions. In [23], the authors proposed an enhanced CCA performed in accordance with the IEEE 802.15.4 standard, which increased throughput by 7%.

In article [24], a new method for the backoff mechanism used in IEEE 802.15.4 is presented. According to the authors, the backoff period is subdivided into two backoffs, i.e. temporary backoff and next temporary backoff. They claim that the probability of a collision is minimized, and that throughput and packet delivery ratio are improved. In [25], the authors designed an efficient CSMA/CA algorithm for IEEE 802.15.4 networks, utilizing the variable backoff exponent to increase efficiency of data transmission, thereby reducing the packet drop rate.

Paper [26] proposed a new backoff mechanism called priority-based binary exponential backoff (PB-BEB), in which the number of CCAs performed is not equal to two. PB-BEB varied the number of CCAs required for channel access, depending on the number of backoffs undergone by the node, i.e. the highest priority is assigned to the node which has undergone more backoffs during earlier time slots while accessing the wireless channel. The authors claim that PB-BEB outperforms the binary exponential backoff method (BEB) employed in the IEEE 802.15.4 standard.

### 3. Problem Statement

Let us consider scenario 1 with a star topology-enabled WSN, with  $N_T$  sensor devices located within the transmission range of a PAN coordinator node. Let A and B denote two devices contending for channel access within the network. At time slot  $t_1$ , let device A be successful in conducting the first CCA (upon finding the idle channel). At time slot  $t_2$ , device A finds the channel busy, after conducting the second CCA. Alternatively, at the same time slot  $t_2$ , device B conducts the first CCA. In this scenario, the IEEE 802.15.4 standard (CCA=2) fails, because device A finds the channel busy while conducting the second CCA. Figure 3 presents scenario 1 considered for the additional CCA.

Scenario 2 considers a star topology-enabled WSN with  $N_T$  sensor devices located within the transmission range of a PAN coordinator node. At time slot  $t_1$ , let device A be successful in conducting the first CCA (upon finding the idle channel). At time slot  $t_2$ , device A finds the channel busy, after conducting the second CCA. Alternatively, at the same time slot  $t_2$ , device B waits for an acknowledgement signal from the PAN coordinator node. In this scenario, the IEEE 802.15.4 standard (CCA=2) fails, because device A finds the channel busy while conducting the second CCA. Figure 4 presents scenario 2 considered for the additional CCA.

### 4. Proposed Methodology

Here, a detailed explanation of the proposed additional CCA algorithm is provided for the two scenarios encoun-



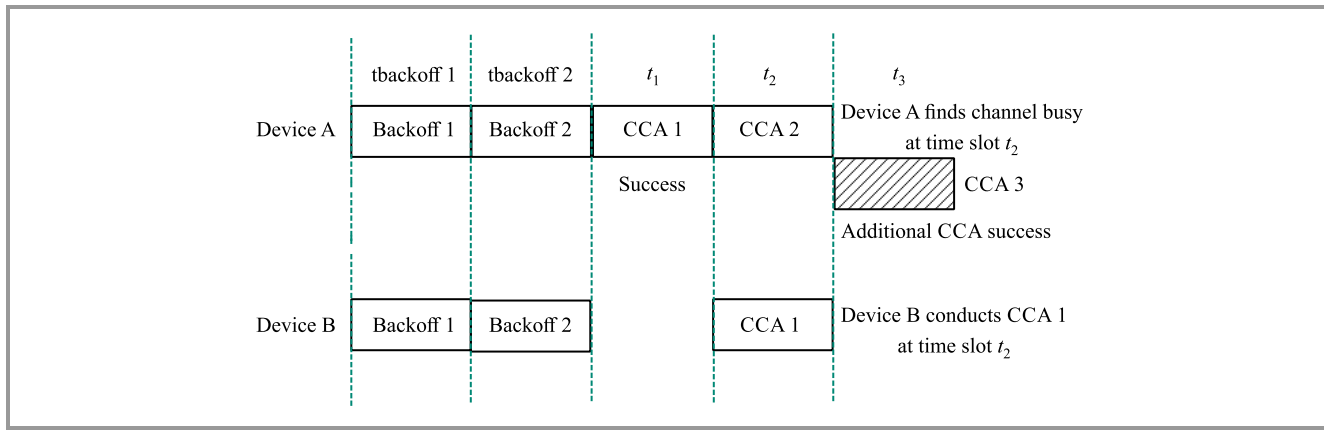


Fig. 3. Scenario 1 considered for additional CCA.

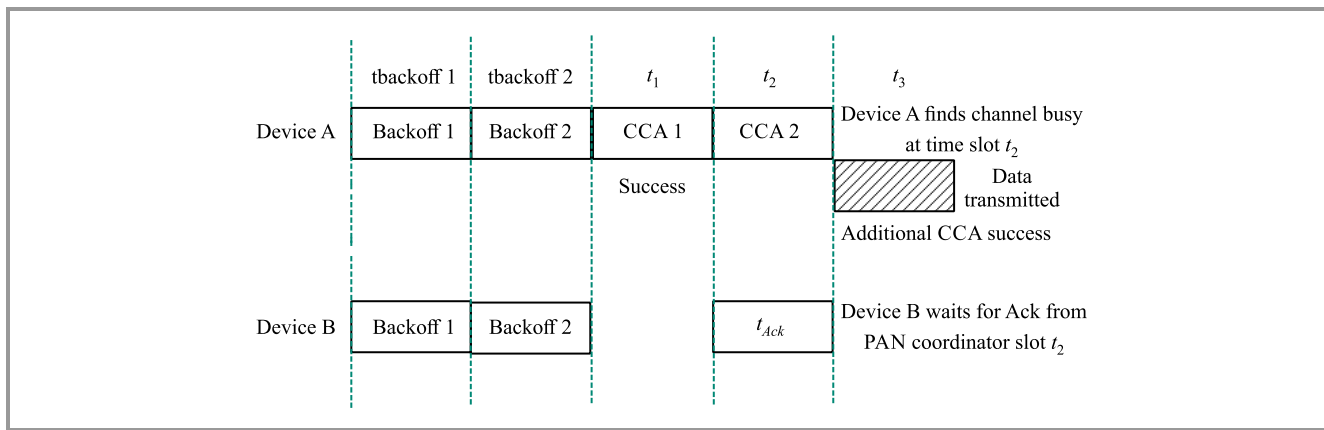


Fig. 4. Scenario 2 considered for additional CCA.

tered in a star topology network. Algorithm 1 is incorporated to enhance performance of the IEEE 802.15.4-based MAC protocol. It assumes that two nodes in a network are contending for a channel and that the highest priority is assigned to the node which has completed multiple backoffs, rather than to the node which has just started to conduct CCA.

The algorithm starts by initializing inputs contention window ( $CW=3$ ), number of backoff stages ( $NB=0$ ), minimum value of backoff exponent ( $macMinBE=3$ ), maximum value of backoff exponent ( $macMaxBE=5$ ), and the number of sensor devices (line 1). Additionally, the time slots required to complete the backoffs and the CCA states (line 2) are determined. Next, the channel access mechanism is set by locating the backoff period boundary (lines 5 and 6). Therefore, it waits for a random period ( $0$  to  $2^{BE} - 1$ ) for certain backoff time slots. The number of backoff time slots is  $t_{backoff1}, t_{backoff2}, \dots, t_{backoffn}$  (line 7).

After a suitable backoff period, at time slot  $t_1$ , Node\_A performs CCA\_1 to check the status of the channel (line 8). Then, the algorithm enters into a loop, checking the status of the channel, and determining it to be either idle or busy. Next, if the channel is found to be idle at the end of its first CCA, the contention window is decremented until it reaches zero ( $CW=0$ ) (line 9). Otherwise, the algorithm jumps to line 15 (where the channel is busy). However, if

$CW=0$ , then Node\_A is set to transmit a data packet and waits for an acknowledgement signal in the network (line 10). Otherwise, CCA\_1 expires and the algorithm moves to line 13. Here, if the channel is found to be busy after completion of CCA\_1, then Node\_A proceeds to conduct CCA\_2, upon finding the channel to be idle, Node\_A is set to transmit a data packet and waits for an acknowledgement signal in the network (lines 16 to 17). CCA\_2 expires after completion of time slot  $t_2$  (line 18). The algorithm declares the channel status as busy due to the failure of CCA\_2 for the two scenarios considered. Furthermore, the algorithm incorporates an additional CCA to address the busy channel condition as mentioned below.

**Case 1.** The performance of an additional CCA (CCA\_3) is described based on the first condition (line 21). After the end of time slot  $t_{backoff2}$ , Node\_A completes CCA\_1 and CCA\_2 at time slots  $t_1$  and  $t_2$ , respectively. Alternatively, Node\_B (node contending for channel access in the network) completes its CCA\_1 at time slot  $t_2$  (same time slot utilized by Node\_A for conducting CCA\_2). Here, at time slot  $t_2$ , CCA\_2 employed by Node\_A fails, since Node\_B is also contending for channel access. To overcome the failure of CCA\_2 in scenario 1, Node\_A is set to perform an additional CCA (CCA\_3) at time slot  $t_3$  (line 22) for successful transmission of the data packet.

**Algorithm 1:** Additional clear channel assessment

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1: Input: Initialize CW=3, NB=0, macMinBE=3,
   macMaxBE=5, Number of sensor devices
2: Input: Initialize time slots  $t_{backoff1}$ ,  $t_{backoff2}$ ,  $t_{backoffn}$ 
    $t_1$ ,  $t_2$ ,  $t_3$ 
3: Output: IEEE 802.15.4 standard optimized parameters
    $P_c$ ,  $P_L$ ,  $P_{ts}$ ,  $T_i$ ,  $T_{col}$ 
4: Begin
5: Set channel access mechanism
6: Locate the backoff period boundary
7: Set random wait duration NB  $\leftarrow$  (0 to  $2^{BE} - 1$ ) at
   time slot  $t_{backoff1}$ ,  $t_{backoff2}$ ,  $t_{backoffn}$ 
8: Set Node_A to perform CCA_1 after waiting for a ran-
   dom duration at time slot  $t_1$ 
9: If channel=idle Then Set CW  $\leftarrow$  CW-1
10:   If CW=0 Then
11:     Set Node_A  $\leftarrow$  Transmission and wait for the
       acknowledgement
12:   Else CCA_1 expires and Goto line 7
13:   Endif
14: Else // in case the channel is busy
15: Set Node_A to perform CCA_2 after successful com-
   pletion of CCA_1 at time slot  $t_2$ 
16:   If channel = idle Then Set CW  $\leftarrow$  CW-1
17:   If CW=0 Then Set Node_A  $\leftarrow$  Transmis-
       sion and wait for the acknowledgement
18:   Else CCA_2 expires and Goto line 7
19:   Endif
20:   Endif
21: If (Node_A CCA_2=failed && Node_B CCA_1=suc-
   cess && channel=busy) Then // case 1
22: Set Node_A to perform CCA_3 (additional CCA) at
   time slot  $t_3$ 
23: Elseif (Node_A CCA_2=failed && Node_B=waiting
   for the ACK && channel=busy) Then // case 2
24: Set Node_A to perform CCA_3 (additional CCA) at
   time slot  $t_3$ 
25: Endif
26: Set CW  $\leftarrow$  3
27: Set NB  $\leftarrow$  NB +1
28: Set BE  $\leftarrow$  min (BE +1, a MaxBE)
29: Endif
30: If (NB > NB Max) Then Stop channel access mecha-
   nism
31: Else Goto line 5
32: Endif
33: End

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**Case 2.** The performance of an additional CCA (CCA\_3) is described based on the second condition (line 23). After the end of time slot  $t_{backoff2}$ , Node\_A completes CCA\_1 and CCA\_2 at time slots  $t_1$  and  $t_2$ , respectively. Alternatively, Node\_B (node contending for channel access in the network) complete its CCA\_2 at time slot  $t_2$  (same time slot as utilized by Node\_A for conducting CCA\_1). Next, Node\_B waits for an acknowledgement signal from the PAN coor-

dinator node in the network. Here, at time slot  $t_2$ , CCA\_2 employed by the Node\_A fails, since Node\_B is also contending for channel access. In order to overcome the failure of CCA\_2 in scenario 2, Node\_A is set to perform an additional CCA (CCA\_3) at time slot  $t_3$  (line 24) for successful transmission of the data packet.

Inputs CW, NB, BE are updated (lines 26–28) after the completion of the additional CCA. Finally, the algorithm checks if the maximum number of backoffs has been exceeded (line 30). If the condition is satisfied, the algorithm stops channel sensing, else it repeats line 31 and performs an additional CCA at time slot  $t_3$  instead of waiting for a random backoff period and thus improves the overall efficiency of the network.

## 5. Analytical Model

A Markov chain model is developed for the proposed additional CCA in order to tackle the two scenarios mentioned in Section 3. The total number of nodes equals  $N_T$ . They are connected forming a star topology network with a PAN coordinator node. The stochastic processes for conducting the additional CCA are modeled using a Markov chain model illustrated in Fig. 5. We consider  $s(t)$  as the number of backoff stages,  $m(t)$  as the contention window,  $n(t)$  as the counter of backoff stages,  $t_{SC}(t)$  as the successful transmit cycle and collision cycle, respectively, where  $s(t) \in \{0, 2\}$ ,  $m(t) \in \{-1, 0, \dots, W_j\}$ ,  $n(t) \in \{0, 1, 2, 3\}$  and  $t_{SC}(t) \in \{0, 1, 2, \dots, L_f\}$ .

$$W_j = 2^{aminBE+i}, \quad 0 \leq i \leq 2. \quad (1)$$

Let

$$S_{i,j,k,l} = \lim_{t \rightarrow \infty} P\{s(t)=i, m(t)=j, n(t)=k, t_{SC}(t)=1\}, \quad (2)$$

where

$$i \in [0, 2], j \in [-1, W_j], k \in [0, 3], l \in [0, L_f].$$

State  $S_{i,j,k,l}$  represents changes that occurred in the channel access mechanism during the additional CCA. States from  $(i, j, k)$  to  $(i, j-1, k)$  represent the number of backoff states. Therefore, during the backoff period, a number of sensor devices in the network are in idle or sleep modes. States from  $(i, j, k)$  to  $(i, j-1, k-2)$  represent the number of CCAs conducted. A number of sensor devices are at this stage during the busy state of a wireless channel. States from  $(i, j, l)$  to  $(i, j, l+1)$  represent a successful transmission of the packet and the probability of a collision. The number of sensor devices will be in a transmission cycle during the idle state of the wireless channel. Also, the probability of a collision represents the number of sensor devices that experienced collisions during transmission of the data packet or its acknowledgement. Let  $x_1$  be the probability condition for CCA during the first channel access attempt,  $x_2$  be the probability condition for conducting CCA during the second channel access attempt caused by the previous

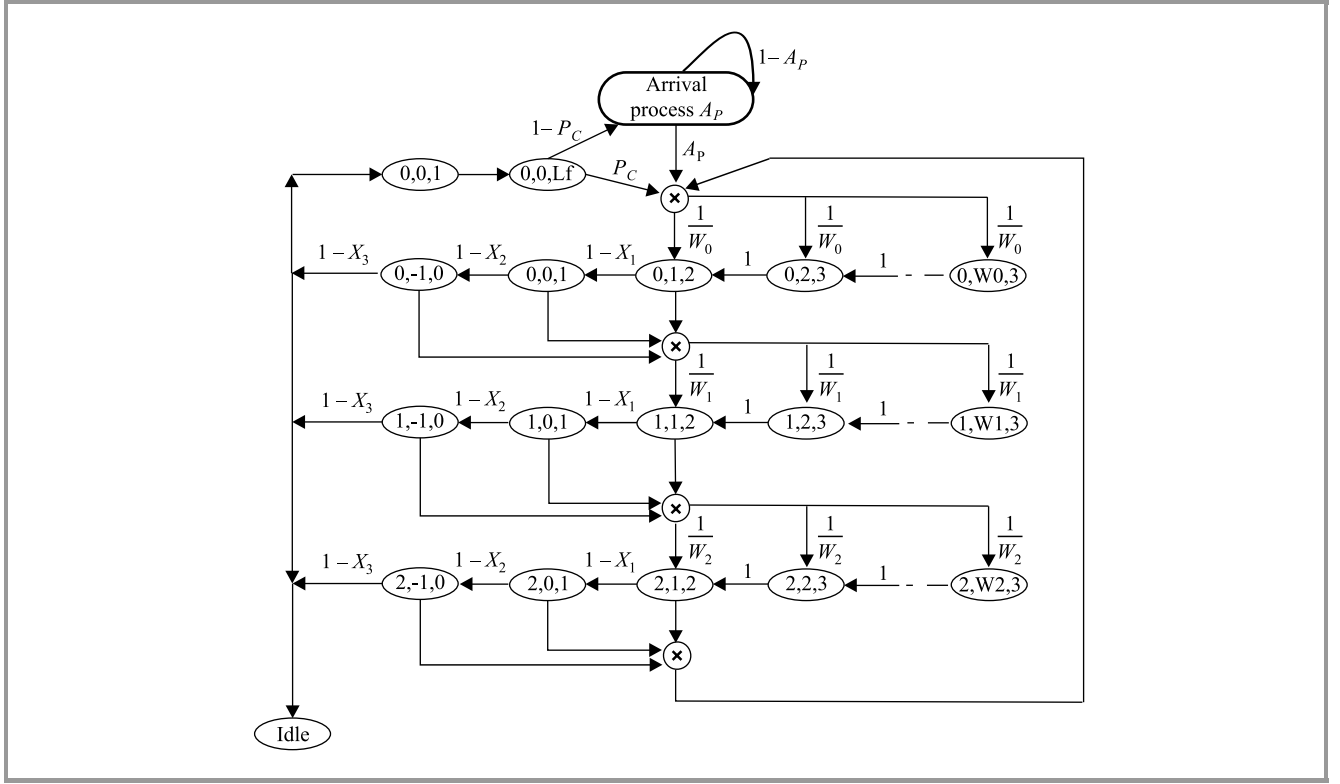


Fig. 5. Markov chain model for additional CCA.

idle channel condition, and  $x_3$  be the probability condition for an additional CCA.

As depicted in Fig. 5, the state transition probabilities are:

$$P\{i, j-1, k|i, j, k\} = 1, \quad j \in [-1, W_j], \quad (3)$$

$$P\{i, j-1, k-2|i, j, k\} = (1-x_1)(1-x_2)(1-x_3), \quad j \in [-1, W_j], \quad k \in [0, 3], \quad (4)$$

$$P\{i, j, l+1|i, j, l\} = (1-x_1)(1-x_2)(1-x_3)(1-P_C), \quad l \in [0, L_f], \quad (5)$$

$$P\{i, j, l+1|i, j, l\} = (1-x_1)(1-x_2)(1-x_3)P_C, \quad l \in [0, L_f]. \quad (6)$$

Equation (3) represents the state transition with probability 1, whenever there is a backoff counter decrement, while Eq. (4) represents the state transition after the additional CCA has been conducted. Equation (5) represents the state transition for a successful cycle and Eq. (6) represents the state transition for a collision cycle. Table 1 provides the list of symbols used in the analytical model.

## 6. Performance Metrics

In this section we define the various performance metrics of the additional CCA, such as probability of collision,

packet loss rate, probability of data transmission, probability of a successful data transmission, channel utilization, channel collision time, channel idle time, reliability, delay, energy consumption of a node, and investigate their impact on the IEEE 802.15.4-based MAC protocol. While conducting the CCA, the node's state changes based on the traffic generated in the network. Channel sensing probability defines probability changes associated with a given node, i.e. the node may be in the idle, collision, busy and transmit state.

The channel sensing probability is:

$$\emptyset = \frac{2}{W_b^{\max} + 1 + K1 + K2}, \quad (7)$$

where

$$K1 = 2L_f(1-x_1)(1-x_2)(1-x_3), \quad (8)$$

$$K2 = 2[(1-x_1) + (1-x_1)(1-x_2)], \quad (9)$$

Whenever nodes contend for wireless channel access, there is a chance of packet collisions in the network. This metric provides details of all nodes which suffered from packet collisions in the network while accessing the channel. The nodes experience packet collisions due to simultaneous data packet transmissions performed by all nodes. Hence, collision probability always increases with the number of nodes in the network and is defined as the probability of all nodes being in the collision state:

$$P_C = 1 - (1-\emptyset)^{N_T-1}. \quad (10)$$



Table 1  
List of symbols used

| Symbol           | Description                                      |
|------------------|--------------------------------------------------|
| $P(i, j, k)$     | State transition probability                     |
| $T_{mean}$       | Average time                                     |
| $T_{busy}$       | Total time during busy cycle                     |
| $T_{bof}$        | Total backoff time                               |
| $T_{cc}$         | Total time for CCA conduction                    |
| $T_{col}$        | Total time during collision cycle                |
| $T_{suc}$        | Total time during success cycle                  |
| $T_{rcv}$        | Total time during receive cycle                  |
| $T_i$            | Time during channel in idle state                |
| $T_a$            | Turnaround time during receive cycle             |
| $T$              | Total time                                       |
| $n_F^{\max}$     | Maximum value for frame retries                  |
| $W_b^{\max}$     | Maximum value for random back off time           |
| $N_T$            | Total nodes in a network                         |
| $N_{act}$        | Total active nodes in a network                  |
| $N_{CCA}$        | Total nodes conducting CCA                       |
| $R$              | Reliability of a network                         |
| $\emptyset$      | Sensing probability of channel                   |
| $\emptyset_{11}$ | Sensing probability of channel in state $S_{11}$ |
| $P_c$            | Probability of collision                         |
| $P_d$            | Probability of data transmission                 |
| $P_{ts}$         | Probability of successful data transmission      |
| $C_{ut}$         | Channel utilization                              |
| $\pi_{suc}$      | Transition for success cycle                     |
| $\pi_{rcv}$      | Transition for receive cycle                     |
| $\pi_{busy}$     | Transition for busy cycle                        |
| $\pi_{col}$      | Transition for collision cycle                   |
| $P_t$            | Power consumed during transmission mode          |
| $P_r$            | Power consumed during receive mode               |
| $P_{cc}$         | Power consumed during CCA conduction             |
| $P_i$            | Power consumed during idle mode                  |
| $P_L$            | Packet loss rate                                 |
| $L_f$            | Length of the frame                              |
| $E_{Tcc}$        | Total energy consumed for CCA computation        |
| $E_{Tb}$         | Total energy consumed for backoff state          |
| $E_T$            | Total energy for all states                      |
| $E_t$            | Total energy consumed for transmitting           |
| $E_r$            | Total energy consumed for receiving              |
| $X$              | Success state                                    |
| $Y$              | Busy state                                       |
| $Z$              | Collision state                                  |

As nodes encounter repeated collisions in the network while accessing the wireless channel, a parameter known as packet loss rate may be defined. Efficiency of the network is degraded if the packet loss rate is high. Therefore,

the packet loss rate determines the number of packets lost in the network and depends on the number of nodes in the network:

$$P_L = [1 - (1 - \emptyset)^{N_T - 1}]^{n_F^{\max} + 1}. \quad (11)$$

The probability of finding any single node transmitting a data packet after the performance of the additional CCA is equal to the probability of data transmission. This metric determines the number of nodes involved in data transmission after successful completion of the additional CCA, without acknowledgment from the receiver. In other words, this metric does not provide any information about the packets lost due to collisions and about the acknowledgement received from the other nodes in the network. Therefore, the probability of data transmission is:

$$P_{td} = 1 - (1 - \emptyset)^{N_T}. \quad (12)$$

The probability of a successful data transmission is expressed as:

$$P_{ts} = \frac{N_T \emptyset (1 - \emptyset)^{N_T - 1}}{P_{td}}. \quad (13)$$

This metric is defined as the probability of a successful transmission of data packets without any collisions in the network. It is determined based on the acknowledgement received by the other nodes in the network.

Effective channel utilization is the most important metric measured in order to determine network efficiency. It is possible to achieve a maximum channel utilization rate in a network if all nodes spend more time transmitting or receiving data packets without any collisions and spend less time on other activities, such as CCA, and in busy states. Let  $N_{act}$  be the number of active sensor nodes in a network, contending for channel access. Then, effective channel utilization  $C_{ut}$  is:

$$C_{ut} = N_{act} L_f \emptyset (1 - \emptyset)^{N_T - 1} (1 - x_1)(1 - x_2)(1 - x_3). \quad (14)$$

In WSNs, it is always better to keep the wireless channel busy with performing any activities, such as data packet transmission or reception between the nodes. All nodes should be busy, either conducting CCA or busy with the backoff states. Therefore, performance of the network is degraded if a channel is in the idle state. Channel idle time is defined as the wireless channel not performing any activities in the network. The idle time parameter should be as low as possible, and it is always a complementary phase in the process of conducting a clear channel assessment (CCA) during the first channel access attempt. It is given by:

$$T_i = 1 - x_1. \quad (15)$$

In WSNs, if there are more nodes contending for access to a given channel, then the probability of a collision will always be high. In order to enhance the channel utilization rate of a network, we need to ensure that the total time spent by all nodes dealing with packet collisions should be as low as possible. The channel collision time

is defined as the total time spent by all nodes in the collision state:

$$T_{col} = x_1 - N_{act} L_f \theta (1 - \theta)^{N_T - 1} (1 - x_1)(1 - x_2)(1 - x_3) . \quad (16)$$

In CCA, the reliability of a network plays a major role in determining network efficiency. Reliability of the additional CCA is determined based upon state transitions occurring in the network while accessing the wireless channel. Whenever a node performs CCA, it enters into three possible states, such as success state, busy state, and collision state. In the success state, the node delivers data packets to the destination node, in the busy state, the node is waiting for the channel, and in the collision state, the node experiences packet collisions. This metric is defined as the probability of a node being in the success state without any packet collisions:

$$R = \frac{1}{1 + \frac{[1 - Y]Y^{m+1}}{[1 - Y^{m+1}]X} + \frac{Z^{n+1}}{((1 - Y)^{n+1}) - z^{n+1}}} . \quad (17)$$

In WSNs, all nodes are battery powered and energy consumption of a node plays a major role whenever an additional CCA is performed in the IEEE 802.15.4-based MAC protocol. Hence, energy consumption of a node is computed based on the data packet transmitted or received by that node, as well as based on the node's state change (i.e. collision or busy state):

$$E_T = E_t + E_r + E_{Tb} + E_{Tcc} , \quad (18)$$

$$E_T = (P_t + P_r)L_f(1 - \theta)^{N_T - 1}(1 - x_1)(1 - x_2)\theta_{11} + P_i \frac{W_b^{\max} - 1}{2} \theta_{11} + P_{cc}[2 - x_1]\theta_{11} . \quad (19)$$

The average delay is defined as the total time needed to complete the successful transmission of the data packet by a node. It determines the overall delay experienced by the node whenever an additional CCA is performed. Therefore, computation of the overall delay of a node is based on its state transitions occurring in the network. Further, we express the overall delay based on the time spent by the node on completing the successful transmission or on receiving a data packet, time spent in the busy state, and time spent in the collision state. The overall delay is:

$$T = T_{suc} + T_{rcv} + T_{busy} + T_{col} . \quad (20)$$

The mean delay is:

$$T_{mean} = \left[ 1 + \frac{\pi_{col}}{\pi_{suc}} + \frac{\pi_{rcv}}{\pi_{suc}} + \frac{\pi_{busy}}{\pi_{suc}} \right] T_{bof} + \frac{\pi_{busy}}{\pi_{suc}} T_{cc} + \left[ 1 + \frac{\pi_{col}}{\pi_{suc}} + \frac{\pi_{rcv}}{\pi_{suc}} \right] [3 + L_f] + \frac{\pi_{rcv}}{\pi_{suc}} T_a . \quad (21)$$

## 7. Performance Evaluation

The performance of the additional CCA for the analytical model developed in Section 5 is evaluated using Matlab software. The impact of additional CCA on some of the performance metrics in the IEEE 802.15.4 based MAC protocol is investigated, and the performance metrics are analyzed. Table 2 provides the simulation parameters used in the model.

Table 2  
Simulation parameters

| Parameter                             | Value         |
|---------------------------------------|---------------|
| Band                                  | 2.4 GHz       |
| Channel bandwidth                     | 250 kbps      |
| Channel number                        | 11            |
| Unit backoff period                   | 20 symbols    |
| Number of nodes                       | 5–500         |
| Turnaround time ( $T_a$ )             | 12 symbols    |
| Transmission power consumed ( $P_t$ ) | 32 mW         |
| Receiving power consumed ( $P_r$ )    | 38 mW         |
| CCA power consumed ( $P_{cc}$ )       | 38 mW         |
| Idle power consumed ( $P_i$ )         | 0.8 mW        |
| macMaxCSMABackoffs                    | 5             |
| macMaxFrameRetries                    | 4             |
| macMinBE                              | 3             |
| macMaxBE                              | 5             |
| Frame length ( $L_f$ )                | 12 time slots |

As depicted in Fig. 6, it is clear that the probability of a collision increases with the number of devices in the network. The probability of a collision for the IEEE 802.15.4 standard is 15.27% at  $N = 5$ , 67.58% at  $N = 100$ , and 98.06% at  $N = 500$ , respectively. Similarly, the probability of a collision for the AS 802.15.4 MAC protocol is 12.20% at  $N = 5$ , 63.54% at  $N = 100$ , and 96.06% at  $N = 500$ , respectively. Finally, the probability of a collision

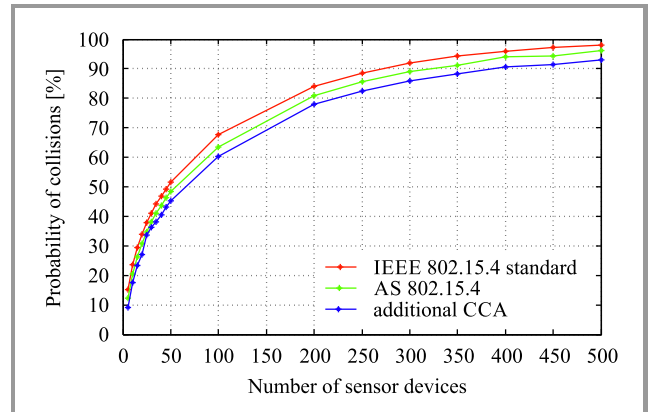


Fig. 6. Probability of collision versus network size.

sion for the additional CCA conducted is 9.16% at  $N = 5$ , 60.32% at  $N = 100$ , and 93.12% at  $N = 500$ , respectively. The probability of a collision is reduced with an additional CCA in comparison with IEEE 802.15.4 and AS 802.15.4 MAC.

Figure 7 presents an evaluation of the packet loss rate parameter. When compared to the IEEE 802.15.4 and AS 802.15.4, the packet loss rate is reduced whenever an additional CCA is performed. It is evident from Fig. 7 that the packet loss rate for IEEE 802.15.4 is 0.05% at  $N = 5$ , 20.85% at  $N = 100$ , and 92.47% at  $N = 500$ , respectively. Similarly, the packet loss rate for the AS 802.15.4 is 0.03% at  $N = 5$ , 17.81% at  $N = 100$ , and 89.45% at  $N = 500$ , respectively. Finally, the packet loss rate for the additional CCA conducted is 0.01% at  $N = 5$ , 13.75% ( $N = 100$ ), and 86.53% ( $N = 500$ ), respectively.

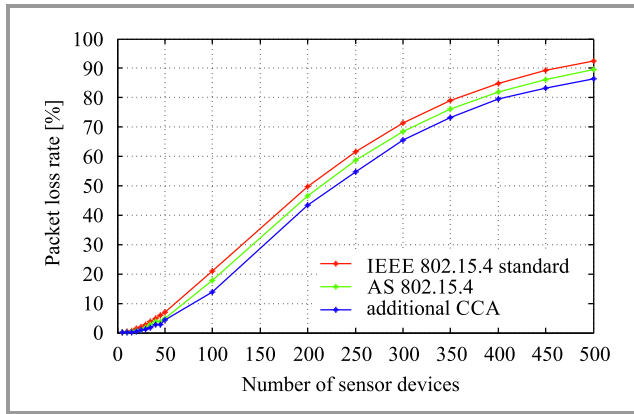


Fig. 7. Packet loss rate versus network size.

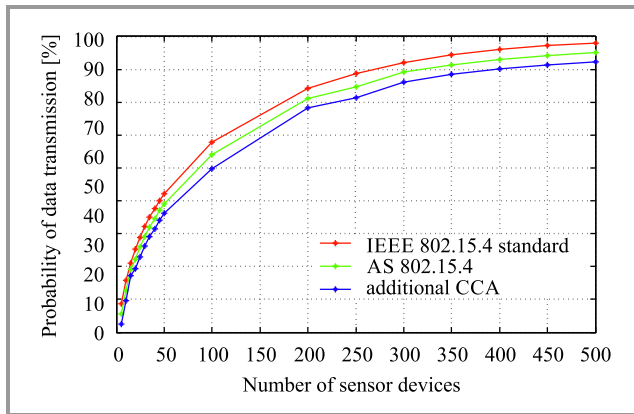


Fig. 8. Probability of data transmission versus network size.

Figure 8 shows an evaluation of data transmission probability. When compared to the existing standards, the probability of data transmission is reduced with an additional CCA. It is observed from Fig. 8 that the probability of data transmission for the IEEE 802.15.4 standard is 18.71% at  $N = 5$ , 67.94% at  $N = 100$ , and 98.08% ( $N = 500$ ), respectively. Similarly, the probability of data transmission for the AS 802.15.4 MAC protocol is 15.63% at  $N = 5$ , 63.91% ( $N = 100$ ), and 95.07% ( $N = 500$ ), respectively.

Finally, the probability of data transmission for the additional CCA is 12.56% at  $N = 5$ , 59.78% ( $N = 100$ ), and 92.32% ( $N = 500$ ), respectively.

Figure 9 presents an evaluation of the probability of successful data transmission. When compared to the IEEE 802.15.4 and AS 802.15.4 MAC protocol, the probability of successful data transmission is significantly enhanced whenever an additional CCA is conducted. The probability of successful data transmission for the additional CCA conducted is 97.87% at  $N = 5$ , 59.17% at  $N = 100$ , and 13.54% at  $N = 500$ , respectively.

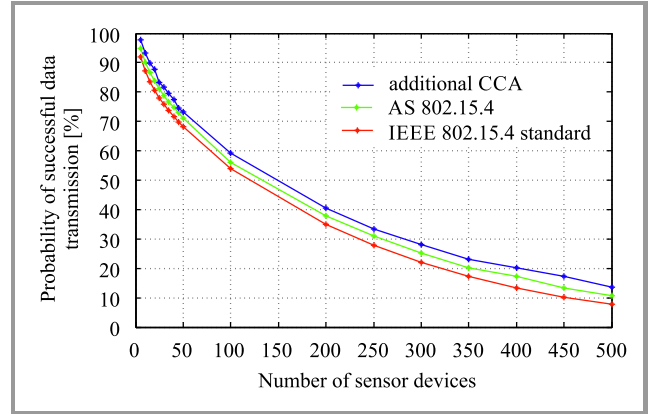


Fig. 9. Probability of successful data transmission versus network size.

As shown in Fig. 10, effective channel utilization is high initially, because of the lower number of nodes contending for channel access. It decreases later on due to the additional nodes being present in the network. It is observed from Fig. 10 that channel utilization with the additional CCA is 69.79% ( $N = 5$ ), 43.48% ( $N = 100$ ), and 3.98% at ( $N = 500$ ), respectively. The channel utilization metric is degraded as the additional CCA is introduced.

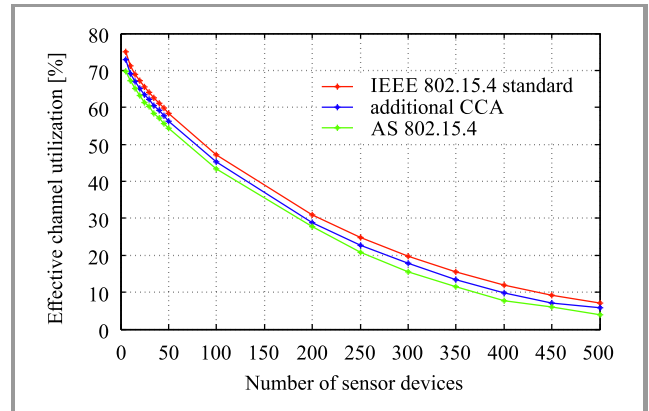


Fig. 10. Effective channel utilization versus network size.

Figure 11 clearly shows that the channel idle time is high initially, because the probability of a channel being in the idle state is higher when compared to later stages, due to the additional nodes in the network. Channel idle time for the additional CCA conducted is 37.99% ( $N = 5$ ), 19.91%

( $N = 100$ ), and 17.74% ( $N = 500$ ), respectively. Channel idle time is higher whenever an additional CCA is conducted in comparison to existing standards.

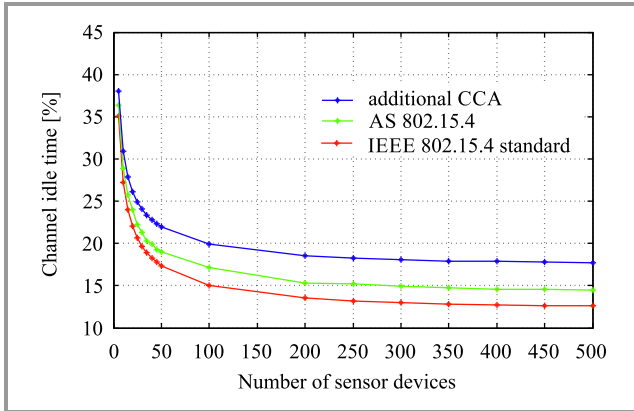


Fig. 11. Channel idle time versus network size.

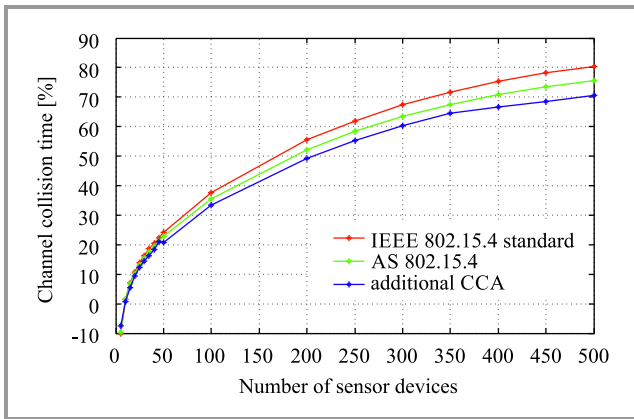


Fig. 12. Channel collision time versus network size.

Channel collision time always increases along with the number of nodes in the network (Fig. 12). Channel collision time for the additional CCA conducted is -7.62% at  $N = 5$ , 33.48% at  $N = 100$ , and 70.71% at  $N = 500$ , respectively. Channel collision time is reduced whenever an additional CCA is performed.

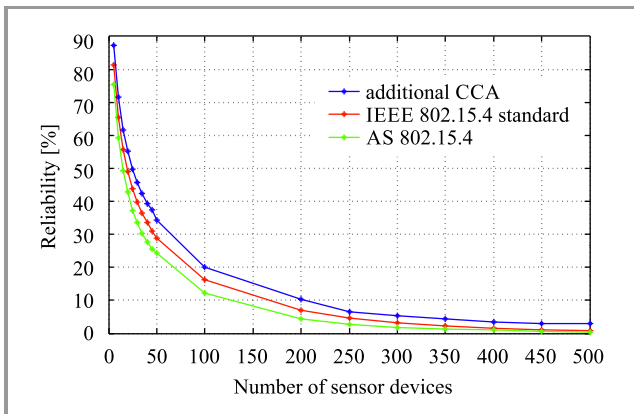


Fig. 13. Reliability versus network size.

As depicted in Fig. 13, reliability of the network is reduced with an increase in the number of nodes. Reliability for the additional CCA is 87.32% at  $N = 5$ , 20.13% at  $N = 100$ , and 2.85% at  $N = 500$ , respectively. The reliability of a network is enhanced with the additional CCA, compared to the IEEE 802.15.4 standard and the AS 802.15.4 MAC protocol.

Energy consumption of a node in the case of the IEEE 802.15.4 standard is 4.1757 J at  $N = 5$ , 0.5189 J at  $N = 100$ , and 0.3684 J at  $N = 500$  respectively (Fig. 14). Similarly, the energy consumption of a node for the AS 802.15.4 MAC protocol is 4.5216 J at  $N = 5$ , 0.6456 J at  $N = 100$ , and 0.4566 J at  $N = 500$ , respectively. Energy consumption of a node for the additional CCA conducted is 4.8122 J at  $N = 5$ , 0.8721 J at  $N = 100$ , and 0.6751 J at  $N = 500$ , respectively. Therefore, energy consumption of the node increases with the additional CCA.

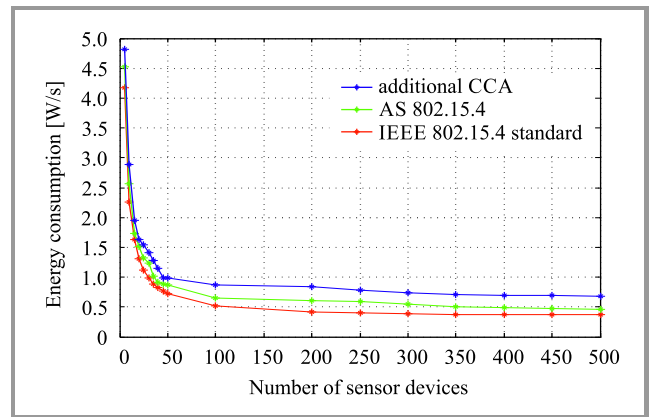


Fig. 14. Energy consumption versus network size.

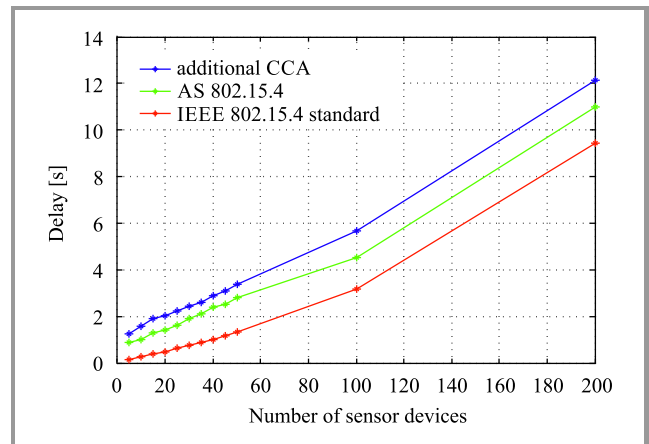


Fig. 15. Delay versus network size.

As shown in Fig. 15, the average delay achieved by a node for the IEEE 802.15.4 standard is 0.1332 s at  $N = 5$ , and 3.1559 s at  $N = 100$ , respectively. Similarly, delay for the AS 802.15.4 MAC protocol is 0.8687 s ( $N = 5$ ), and 4.5403 s ( $N = 100$ ), respectively. Delay for the additional CCA conducted is 1.2578 s at  $N = 5$ , and 5.6534 s

at  $N = 100$ . One may notice that higher delays are caused by conducting the additional CCA.

Table 3

Comparative analysis of existing and proposed standards

| Parameter average (relative)                | IEEE 802.15.4 standard | AS 802.15.4 standard | Additional CCA |
|---------------------------------------------|------------------------|----------------------|----------------|
| Probability of collision                    | 60.56%                 | 57.59%               | 54.66%         |
| Packet loss rate                            | 32.134%                | 30.25%               | 28.58%         |
| Probability of data transmission            | 61.41%                 | 58.33%               | 55.36%         |
| Probability of successful data transmission | 53.77%                 | 56.80%               | 59.66%         |
| Effective channel utilization               | 45.55%                 | 41.60%               | 43.57%         |
| Channel idle time                           | 18.14%                 | 19.88%               | 22.69%         |
| Channel collision time                      | 36.29%                 | 34.17%               | 32.19%         |
| Reliability                                 | 27.87%                 | 23.87%               | 32.25%         |
| Energy consumption                          | 0.9933 J               | 1.1605 J             | 1.3691 J       |
| Delay                                       | 1.6322 s               | 2.7906 s             | 3.4256 s       |

Table 3 presents a comparative analysis of all performance metrics investigated for the IEEE 802.15.4 standard, the AS 802.15.4 MAC protocol, and the additional CCA conducted. The average values of all performance metrics are considered. Furthermore, simulation results show a difference between the IEEE 802.15.4 standard, the AS 802.15.4 MAC protocol, and the additional CCA. From Table 3, it is evident that some of the performance metrics are improved in the IEEE 802.15.4 standard-based MAC protocol by conducting an additional CCA.

## 8. Conclusion

In this paper, we have modeled and analyzed an additional CCA incorporated in the IEEE 802.15.4-based MAC protocol, in a star topology-enabled WSNs. The proposed method addresses the busy channel status encountered in two scenarios considered during the channel access mechanism. In both scenarios, the proposed method relies on an additional CCA in order to enable collision-free data transmission to be performed by all nodes in the network. Initially, a Markov chain model and a set of mathematical equations were developed for the proposed method. Later, we investigated the impact of the additional CCA on certain performance metrics in the IEEE 802.15.4-based MAC protocol. It could be observed that some of the performance metrics regarding the IEEE 802.15.4 standard improved with the introduction of the additional CCA. Simulation results show that there is a significant improvement in performance metrics, such as probability of collision,

packet loss rate, probability of successful data transmission, channel idle time, channel collision time, and the network reliability.

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# EARPC – Energy Aware Routing Protocol for Cooperative MIMO Scheme in WSNs

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<https://doi.org/10.26636/jtit.2020.142720>

**Abstract**—Wireless sensor networks are typically operated on batteries. Therefore, in order to prolong network lifetime, an energy efficient routing algorithm is required. In this paper, an energy-aware routing protocol for the co-operative MIMO scheme in WSNs (EARPC) is presented. It is based on an improved cluster head selection method that considers the remaining energy level of a node and recent energy consumption of all nodes. This means that sensor nodes with lower energy levels are less likely to be chosen as cluster heads. Next, based on the cooperative node selection in each cluster, a virtual MIMO array is created, reducing uneven distribution of clusters. Simulation results show that the proposed routing protocol may reduce energy consumption and improve network lifetime compared with the LEACH protocol.

**Keywords**—cluster head, cooperative MIMO, virtual MIMO, wireless sensor networks.

## 1. Introduction

One of the major concerns affecting WSNs is to minimize energy consumption for a single end-to-end transmission and to improve network lifetime. Over the years, various techniques have been proposed to improve energy efficiency in the energy-constrained environment. Among these approaches, multi-input and multi-output (MIMO) techniques, using more than one antenna, may be considered an effective solution for energy saving in WSNs. By considering various space-time coding scenarios and architectures, the MIMO approach is capable of improving channel capacity and of further reducing energy consumption [1]–[4]. However, multiple antennas cannot be connected to a single transmitter, and the antenna array cannot be accommodated in a sensor node due to the fixed frequency range.

In this paper, the location of sensor nodes and the remaining energy of nodes are considered while selecting cluster heads, and co-operative nodes are chosen for the MIMO system. This scheme may efficiently balance the load condition of the network and further enhance its lifetime.

In this paper, we propose an energy-aware routing protocol for cooperative MIMO scheme (EARPC), which addresses three limitations of the well-known low-energy adaptive

clustering hierarchy (LEACH) protocol [5]. In LEACH, the selection of cluster heads is performed in the same manner for all nodes. Any node may become a cluster head, regardless of its remaining energy level. To enhance network lifetime, a novel cluster head selection technique that balances energy consumption based on the remaining energy of a node and on recent consumption of energy for all nodes is proposed. This scheme may further reduce the chance of low energy sensor nodes becoming cluster heads. In addition, co-operative nodes are chosen to form a virtual MIMO paradigm, based on the nodes' residual energy. Lastly, we propose an energy consumption model which aims to estimate the amount of energy required for collecting data among the nodes, for aggregating data at the CH level, as well as for intra and inter-cluster communication in the MIMO scheme.

This paper is organized as follows. Section 2 discusses the related work. In Section 3, the EARPC model is described. The energy consumption model is presented in Section 4. In Section 5, results of simulation parameters are presented and discussed. Finally, the paper is summarized in Section 6.

## 2. Related Work

To achieve energy-efficient cooperative MIMO networks, Cui *et al.* proposed, in [6], an energy efficiency and delay metric performance parameter for virtual MIMO approach for a single-hop system, which reduces energy consumption and delay for a given transmission range. In paper [7] Maadani *et al.* showed an adaptive data rate space-time coding (STC) technique. It is introduced for IEEE 802.11-based soft-real-time WSNs in which an enhanced distributed channel access (EDCA) is used at the medium access control (MAC) layer and MIMO transceivers are used at the PHY layer for minimizing average packet delay. Jayaweera *et al.* [8] proved that a precise model for energy consumption and cooperative MIMO technology may be considered as energy-saving for extra overhead. Sathian *et al.* in [9] proposed a trustworthy energy-efficient MIMO (TEEM) routing algorithm for WSNs, re-



ducing energy consumption and enhancing the lifetime of a sensor network. The game theory is also used therein to select cluster heads based on the remaining energy of a node and on the trust level during the cluster head selection process.

Zuo *et al.* [10] proposed a BLAST code based on V-layered space-time for the cooperative communication scheme. This scheme achieves high energy efficiency and does not require any data exchange processes. Cooperative communication and data fusion approaches may further reduce energy consumption by removing data redundancy between sensor nodes [11]. Reddy *et al.* proposed, in [12], a QoS-oriented and energy-efficient routing protocol for cooperative MIMO-based mobile WSN: Q-E2RPC. In this scheme, a single mobile sink is used to reduce energy consumption. Li *et al.* proposed an energy-efficient cooperative MIMO scheme in [13], which combines the energy-efficient LEACH protocol and cooperative MIMO. In this protocol, the location of sensor nodes and the remaining energy of the node are considered while selecting cluster heads, and cooperative nodes are chosen for the MIMO system. This scheme may efficiently balance the load condition of the network and further enhance network lifetime.

### 3. System Model

Let us consider a real-time scenario in which a large number of sensor nodes is installed across the field.  $N$  clusters are formed by dividing the total number of sensor nodes, for collection and transmission of data in each round. Based on this assumption, we define the system model as follows:

- all sensor nodes collect data in the sensing area and transmit the collected data to the cluster head node,
- data aggregation may be performed at the cluster head level, which may save the transmitting energy by removing redundant data. Then, CH nodes forward their data to co-operative nodes,
- finally, co-operative nodes may form a virtual MIMO antenna array and forward their data to the sink node

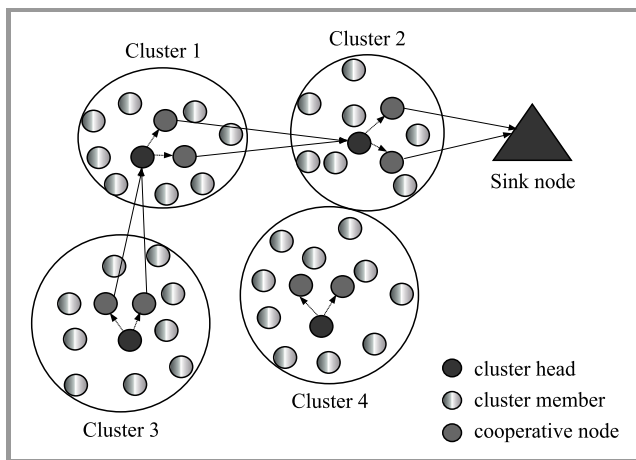


Fig. 1. Schematic representation of data transmission.

using multi-hop communication. An illustration of data transmission is shown in Fig. 1.

The main limitation of the LEACH protocol is that the cluster head selection probability is the same for all sensor nodes. Since the sensor nodes will have different energy levels and consumption rates, if nodes with low remaining energy or faster energy consumption are selected as cluster heads, then CH will stop functioning quickly and will disrupt communication between other nodes in that cluster and the CH. Network lifetime is also reduced if sensor nodes fail early.

To overcome the above-mentioned drawbacks, in this algorithm, the probability of a node becoming a cluster head is based on the ratio of remaining energy of the  $i$ -th node to the recent energy consumption of all nodes.

$$P(i) = \begin{cases} \frac{n}{N - n \left[ r \bmod \left( \frac{N}{n} \right) \right]} \cdot \frac{E_{rem}(i)}{E_{a_{rem}}} \cdot \frac{E_{a_{cons}}}{E_{cons}(i)} & \text{if } i \in G \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

In Eq. (1)  $E_{rem}(i)$  describes the remaining energy of  $i$ -th node and  $E_{a_{rem}}$  refers to average remaining energy of the network. The other parameter  $E_{a_{cons}}$  represents average energy consumption of the last round of transmission in the whole network and  $E_{cons}(i)$  defines energy consumption of the  $i$ -th node.

The second limitation of the LEACH protocol is that the selection of cluster heads is random and the node selected may not be suitable (in terms of energy savings) for sending the collected data to the sink node. To overcome this, a cooperative MIMO routing algorithm is proposed.

After the formation the clusters, a few nodes may be chosen as co-operative nodes to form a virtual MIMO array. Here, the residual energy and distance are taken as important reference points for the selection of co-operative nodes [14]. The co-operative node selection threshold may be obtained from:

$$\delta = \frac{E_{rem}(i)}{d_i}, \quad (2)$$

where  $d_{min} \leq d_i \leq d_{max}$ .

In Eq. (2),  $E_{rem}(i)$  refers to remaining energy of  $i$ -th node and  $d_i$  signifies distance between the cluster head and co-operative nodes, while  $d_{min}$  and  $d_{max}$  refer to minimum and maximum distance, respectively. The cluster head node is responsible for the selection of co-operative nodes with the highest threshold of  $\delta$ , from all the participating nodes.

After co-operative nodes have been identified by the cluster head node based on the above criteria, the cluster head node will send a message to all cooperative nodes requesting their responsibility in virtual MIMO communication. This message contains the ID of co-operative nodes and TDMA schedules are assigned to all nodes.

Figure 1 shows a virtual MIMO data transmission in WSNs. Initially, the cluster head node broadcasts its message to collect data from its members. All sensor nodes forward their sensed data to the corresponding cluster head node

based on their preassigned schedule slots. Then, sensor nodes will go to sleep mode to save energy. Next, cluster head nodes will perform data aggregation to reduce data redundancy (if any). Later, cluster head nodes forward their data to their cooperative nodes in their cluster. This phase is known as intra-cluster communication. Finally, after receiving data from cluster heads, the co-operative nodes will form a virtual MIMO to perform a space-time block code. According to the routing table, the nodes transmit their data to the sink node using multi-hop transmission. This process is called inter-cluster communication.

## 4. Energy Consumption Model

In this section, we propose a model depicting energy consumption during intra- and inter-cluster communication in the network.

### 4.1. Energy Consumption Between the Nodes

The amount of energy consumed depends primarily on energy consumption of the transmitter and the receiver [2], [15]. Power consumption  $P_c$  may be expressed by:

$$P_c = M_t P_{ct} + M_r P_{cr}, \quad (3)$$

where:

$$\left. \begin{aligned} P_{ct} &= P_{DAC} + P_{mix} + P_{filt} + P_{syn} \\ P_{cr} &= P_{LNA} + P_{mix} + P_{IFA} + P_{filr} + P_{ADC} + P_{syn} \end{aligned} \right\}. \quad (4)$$

In Eq. (3),  $M_t$  and  $M_r$  represent the total number of transmitting and receiving nodes, and  $P_{ct}$  and  $P_{cr}$  refer to circuit power consumption of the transmitter's and receiver's circuits. In Eq. (4)  $P_{DAC}$ ,  $P_{mix}$ ,  $P_{filt}$ ,  $P_{syn}$ ,  $P_{LNA}$ ,  $P_{IFA}$ , and  $P_{ADC}$  represent power consumption values of digital to the analog converter (DAC), mixer circuit, filter, frequency synthesizer, low noise amplifier, intermediate frequency, and analog to digital converter (ADC).

The power consumption of power amplifiers  $P_{pa}$  [16] is:

$$P_{pa} = (1 + \alpha) \frac{(4\pi)^2 d^\beta M_l N_f}{G_t G_r \lambda^2} \times \overline{E_b} R_b. \quad (5)$$

In Eq. (5),  $\alpha$  represents the power factor of the amplifier,  $d$  refers to the average distance between the cluster head and cluster members,  $M_l$  and  $N_f$  define link margin and receiver noise,  $\lambda$  is carrier signal wavelength, while  $G_t G_r$  represent the gain of transmitting and receiving antennas.  $\beta$  is the path loss slope,  $\overline{E_b}$  denotes energy consumption required by the receiver (i.e. sink node) to capture each bit of data under certain bit error rate (BER) conditions, and  $R_b$  denotes the data transmission rate under MQAM ( $R_b = \text{bB}$  when the modulation order  $b > 2$ ). The energy required to transmit and receive (per bit) between the nodes is given by:

$$E_{btr}(d) = \frac{P_{pa} + P_c}{R_b} = \frac{P_{pa} + M_t P_{ct} + M_r P_{cr}}{R_b}. \quad (6)$$

### 4.2. Energy Consumption Model within Cluster

Local intra-cluster communication is based on BPSK modulation and uses a single antenna (SISO) for transmission. Let  $P_b$  be denoted as BER [17], [18]. Then, the average energy required to receive a bit correctly is:

$$\overline{E_B^{SI}} = \frac{N_0}{(1 - 2\overline{P_b})^{-2} - 1}, \quad (7)$$

where  $\overline{E_B}$  in Eq. (5) and  $SI$  refer to a single transmitting antenna. With  $M_t = 1$  and BPSK modulation ( $b = 1$ ), energy consumed for intra-cluster communication of 1 bit is:

$$E_{btr}^{SI}(d) = E_{btr}(d) \Big|_{M_t=1 \text{ and } \overline{E_b}=\overline{E_b^{SI}}}. \quad (8)$$

Within each period, every sensor collects  $k$ -bits of data and sends that information to the corresponding cluster head. The total number of sensor nodes  $n_i$  of cluster  $i$  is ( $i = 1, 2, \dots, n$ ), where  $d_{toCH}(i, j)$  represents the distance between the cluster head and intra node. Data collection-related energy consumption may be expressed as [17]:

$$E_{cot}(i) = L \sum_{j=1}^{N_i-1} E_{btr}^{SI}[d_{toCH}(i, j)] \Big|_{M_r=1}. \quad (9)$$

In each transmission round, the cluster head will receive  $L(n_i - 1)$  bits of data of  $i$ -th cluster. Assuming that  $E_{da}$  represents data aggregation-related energy consumption (on a per bit basis), energy consumption related to data aggregation may be given by:

$$E_{agg}(i) = E_{da} L(n_i - 1). \quad (10)$$

The length of data after data aggregation, for cluster head, is:

$$L_f(i) = \frac{L(n_i - 1)}{f_{aff}(n_i - 1) - f_{agg} + 1}, \quad (11)$$

where  $f_{agg} \in (0, 1)$  is the data aggregation factor [18].

### 4.3. Energy Consumption for Intra-cluster Communication

After performing data aggregation, the cluster head node broadcasts  $L_f(i)$  bits of data to co-operative nodes  $N_c$ . To ensure that all sensor nodes in a cluster receive the data correctly, it defines the maximum distance between the cluster head and co-operative nodes by:

$$d_{toCN}(i) = \max \{d_{toCH}(i, j) \mid j \in S_{coop}(i)\}. \quad (12)$$

In Eq. (12),  $S_{coop}(i)$  represents a set of cooperative nodes. Intra-cluster energy consumption may be expressed as [17]:

$$E_{broadcast}(i) = L(i) E_{btr}^{SI}[d_{toCN}(i)] \Big|_{M_r=N_c}. \quad (13)$$

#### 4.4. Energy Consumption for Inter-cluster Communication

During inter-cluster communication, the MIMO technique is used. When BER is less than  $\overline{P}_b$ , the required energy per bit  $\overline{E}_b^{MI}$  is:

$$\overline{E}_b^{MI} = \frac{2}{3} \left( \frac{\overline{P}_b}{4} \right)^{\frac{1}{M_t}} \cdot \frac{2^b - 1}{b^{M_t+1}} \cdot M_t M_0, \quad (14)$$

where  $MI$  denotes multiple inputs i.e. multiple transmitting antennas. Then, the required energy consumption of co-operative nodes communicating between different clusters is [18], [19]:

$$E_{btr}^{MI}(d) = \frac{R_b^{eff}}{R_b} \left[ E_{btr}(d) \Big|_{\overline{E}_b = \overline{E}_b^{MI}} \right], \quad (15)$$

where  $R_b^{eff}$  is the effective bit rate of the system [17]–[19]:

$$R_b^{eff} = \frac{F - pM_t}{F} RR_b, \quad (16)$$

where  $F$  is the block size of space-time block code (STBC),  $p$  denotes the number of symbols used to train each transmitting and receiving antenna pair, and  $R$  denotes the STBC coding rate.

Assuming that cluster  $i$  transmits the data  $h_i$  ( $h_i \geq 1$ ) times to reach the sink node, the energy consumption of such a multi-hop transmission is:

$$E_{mulohop}(i) = L_f(i) \left[ E_{btr}^{MI}[d_{hop}(i, h_i)] \Big|_{M_t=N_c, M_r=1} + \sum_{k=1}^{h_i-1} E_{btr}^{MI}[d_{hop}(i, k)] \Big|_{M_t=M_r+N_c} \right]. \quad (17)$$

In Eq. (17),  $d_{hop}(i, k)$  denotes the distance of each hop ( $i = 1, \dots, h_i$ ) and  $E_{btr}^{MI}$  originates from Eq. (15). Finally, the total energy consumption model  $E_{tot}$  may be expressed using Eqs. (9), (10), (13) and (17) is:

$$E_{tot} = \sum_{i=1}^n [E_{col}(i) + E_{agg}(i) + E_{broadcast}(i) + E_{mulhop}(i)]. \quad (18)$$

## 5. Results and Discussion

This section discusses primarily the experimental set and the results obtained. The performance of EARPC has been evaluated in Matlab, using the existing system, i.e. the LEACH protocol [5], in terms of average residual energy and network lifetime. The network parameters considered for the network model are shown in Table 1. During the simulation, 100 nodes were randomly distributed throughout the area of  $100 \times 100$  m area as shown in Fig. 2. Figure 3 shows the results of average residual energy simulations for both LEACH and EARPC protocols. The residual energy in the LEACH protocol depletes faster than in EARPC.

Figure 4 shows the number of nodes that are alive in each round, for both algorithms considered. Simulation results show that the node survival rate of the proposed protocol

Table 1  
Parameters using in simulation

| Parameters                                          | Value                     |
|-----------------------------------------------------|---------------------------|
| Network area                                        | $100 \times 100$ m        |
| Total number of sensor nodes $N$                    | 100                       |
| Initial energy of network $E_0$                     | 50 J                      |
| Packet size $L$                                     | 2000 bits                 |
| Energy dissipation in power amplifier $E_{amp}$     | 100 pJ/bit/m <sup>2</sup> |
| Energy dissipation during aggregation $E_{DA}$      | 20 nJ/bit                 |
| Energy dissipation in electronics $E_{elec}$        | 50 nJ/bit                 |
| Minimum distance $d_{min}$                          | 1 m                       |
| Maximum distance $d_{max}$                          | 50 m                      |
| Gain of transmitting and receiving antenna $G_r$    | 5 dBi                     |
| Power factor of an amplifier $\alpha$               | 0.47                      |
| Carrier wavelength $\lambda$                        | 0.12 m                    |
| Noise coefficient of the receiver $N_f$             | 10 dB                     |
| Channel bandwidth $B$                               | 10 kHz                    |
| Aggregation factor $f_{agg}$                        | 0.7                       |
| Power of the low-noise amplifier $P_{LNA}$          | 20 mW                     |
| Power of the frequency synthesizer $P_{syn}$        | 50 mw                     |
| Power of the mixer $P_{mix}$                        | 30 mW                     |
| Power of transmitting circuit filter $P_{filt}$     | 2.5 mW                    |
| Power of receiving circuit filter $P_{filr}$        | 2.5 mW                    |
| Power of analog to digital converter $P_{ADC}$      | 10 mW                     |
| Power of digital to analog converter $P_{DAC}$      | 10 mW                     |
| Power of intermediate frequency amplifier $P_{IFA}$ | 20 mW                     |
| Set of symbols $F$                                  | 200                       |
| Bit error rate $\overline{P}_b$                     | $10^{-3}$                 |

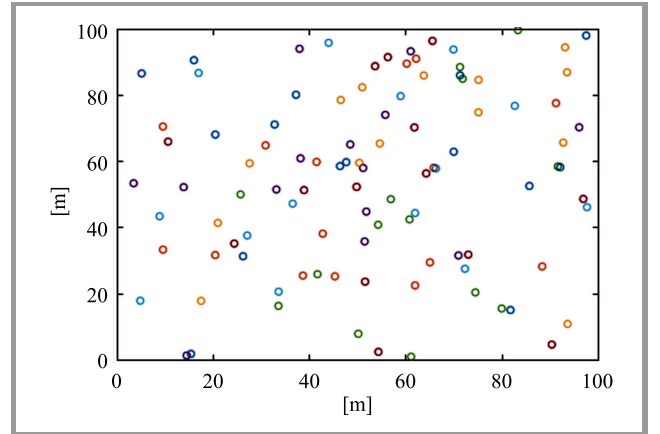


Fig. 2. Distribution of sensor nodes in network area.

is higher than in the case of LEACH. It may be seen from Fig. 4 that the difference starts to become apparent at 1000 rounds, as the number of rounds increases with respect to the number of nodes. At  $r = 1500$  rounds, the survival rate of the LEACH protocol is about 5 and in the case of the proposed protocol it equals approximately 95. So, it is clearly shown that the proposed EARPC protocol is more effective in terms of reducing energy consumption.

Figure 5 presents network lifetime for both LEACH and EARPC protocols. In LEACH, the first node is dead at round 1000, while in EARPC it is at 1550. Similarly, when

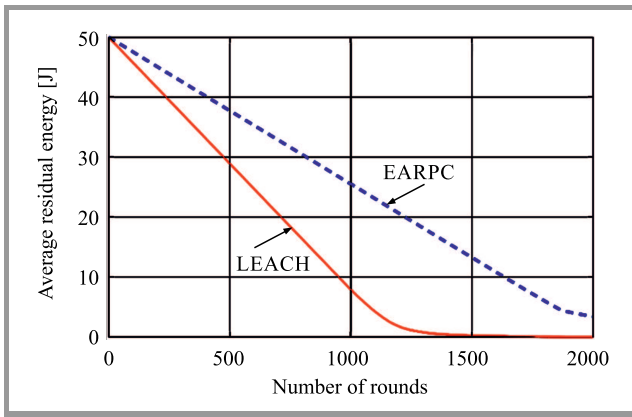


Fig. 3. Average residual energy of sensor nodes.

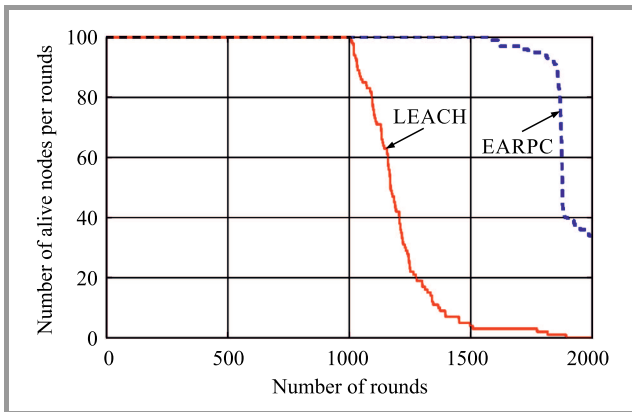


Fig. 4. Total number of alive nodes per round.

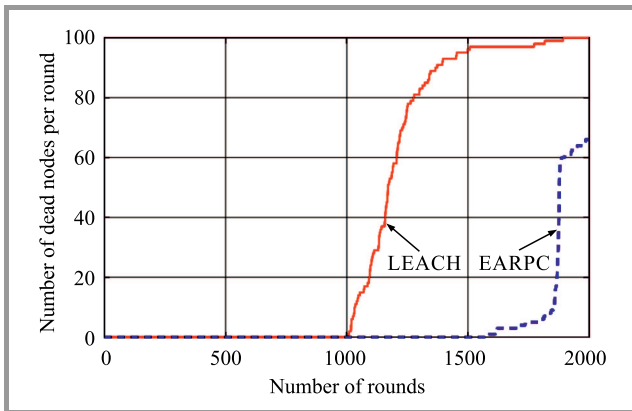


Fig. 5. Number of dead nodes per round.

$r = 2000$ , the share of dead nodes is 100% in LEACH, whereas in EAPRPC, it is about 65%. As proven by the results, the proposed protocol reduces the death rate by 35% compared with the conventional LEACH protocol.

## 6. Summary

The proposed energy consumption model offers better performance in terms of energy consumption in WSNs. The new CH selection technique used balances energy consumption for different sensor nodes in a given cluster. The

cooperative MIMO scheme further reduces uneven CH distributions. Therefore, with the overall performance taken into consideration, the proposed routing protocol is capable of reducing energy consumption and improving network lifetime.

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# ICBCA – Improved Cluster Based Channel Allocation in Cognitive Radio Sensor Networks

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<https://doi.org/10.26636/jtit.2020.139820>

**Abstract**—Wireless sensor networks (WSNs) operate in an overcrowded electromagnetic environment, as the spectrum is shared by various wireless communication technologies. This gives rise to various challenges related to optimized and efficient spectrum utilization. Cognitive radio (CR) has emerged as a solution satisfying this requirement, as it is capable of adapting to the dynamic radio spectrum. Thanks to the deployment of cognitive radio in WSNs, the spectrum may be utilized in a more efficient manner. CR may identify the vacant channels dynamically, allowing the sensor nodes to effectively communicate with each other. In this paper a clustering algorithm known as improved cluster-based channel assignment (ICBCA) is implemented, forming clusters of CR sensor nodes and then selecting vacant channels for data transmission purposes. Simulation results show that ICBCA outperforms existing clustering algorithms in CR sensor networks.

**Keywords**—channel interference, channel overlapping, co-channel interference, cognitive radio.

## 1. Introduction

Wireless sensor networks (WSNs) are a collection of battery powered nodes used to monitor the environment. These networks are successfully applied in monitoring health, environment and habitat monitoring, in military, surveillance and weather applications, as well as in disaster management [1], [2]. All sensor nodes forward the data to the base station (BS). WSNs are often deployed in usually unattended areas where physical presence of human beings is difficult to ensure due to practical reasons. Therefore, the replacement of batteries used by such networks becomes a major energy preservation-related concern [1]. Grouping sensor nodes into clusters may be a remedy enabling to preserve energy in WSNs [3]. Here, the sensor nodes of a network are controlled by a centralized coordinator, known as the cluster head (CH). All nodes within the cluster communicate with the CH forwarding the sensed data. The CH, in turn, communicates with its neighboring CH and this process is continued until the sensed data reach the BS. Research focusing on energy conservation in WSNs is presented in [4] and [5].

Due to advances in technology, more wireless solutions are becoming available everyday, resulting in the excessive

used of the spectrum. The radio spectrum is divided into a licensed band, allocated to licensed users referred to as primary users (PUs), and an unlicensed band allocated to non-licensed users, referred to as secondary users (SUs) [6]. According to a report by the US Federal Communications Commission (FCC), a considerable portion of the radio spectrum is underutilized and there are some vacant spaces, referred as white spaces, which may be efficiently utilized by SUs for communication purposes, as the average spectrum utilization rate ranges from 15% to 85% [7]. Cognitive radio (CR) is a technology that is capable of detecting and using white spaces. It has emerged as a remedy ensuring effective utilization of the spectrum without intervening with the operations of PUs [7], [8].

If CR may be integrated with WSNs, it will be able of overcoming numerous challenges typical of conventional WSNs. The new network paradigm thus formed is called cognitive radio sensor networks (CRSNs) [9]. CR has the ability of knowing the spectrum used in the licensed and unlicensed band, and of utilizing its unused portion in an opportunistic manner. PUs are authorized to use the spectrum at any time, while SUs may utilize the spectrum only in the absence of PUs. The activity of PUs is dynamic [10], hence SUs must be alert enough to track the time periods over which the spectrum is allocated to PUs. However, the SUs may communicate with other SUs only when the spectrum is available. Therefore, CRSN imposes a spectrum awareness constraint. By being aware of spectrum utilization, the nodes may avoid communicating using the crowded licensed band, thereby utilizing the radio spectrum in a more efficient manner [9], [10].

Apart from sensing the surrounding environment, CRSN consumes energy in several different ways, such as:

- sensing the radio spectrum to detect PU presence,
- identifying data transmission and thereby routing data packets,
- to performing channel negotiation.

The rest of the paper is structured as follows: in Section 2, related work on clustering in CRSN, along with their potential drawbacks, are presented. In Section 3, the proposed



protocol is explained. Performance analysis of the proposed protocol is performed by comparing it with existing clustering protocols, such as DSAC [11] and CogLEACH [12] in Section 4. Finally, Section 5 presents the conclusion.

## 2. Related Work

In [11], the authors propose a spectrum-aware clustering algorithm referred to as the distributed spectrum-aware clustering (DSAC) algorithm for CRSN. The algorithm performs clustering by sensing the vacant channels, and thus changes in PU activity. DSAC uses group-wise constrained clustering to minimize energy consumption of CRSN nodes. As the number of active PU nodes increases, more spectrum-aware constraints are imposed on the process of clustering. Hence, DSAC offers poor performance in terms of energy consumption while clustering.

In [12], an extension of the LEACH protocol, known as CogLEACH, suitable for cognitive environment is presented. Each sensor node sends channel state data to the base station. The expected number of clusters is determined by checking the number of nodes required to cover the area, and by verifying the availability of channels at that node. The cluster head (CH) is chosen based on probability value of high availability of channels. Nodes with a high probability value are chosen as CH. Upon formation of clusters, CH nodes generate TDMA schedules and all cluster members start to transmit data within their allotted timeslots. CogLEACH achieves better throughput and lifetime when compared with the traditional LEACH protocol.

In [13], the authors propose an event-driven spectrum-aware clustering algorithm (ESAC) which forms clusters based on event detection. The algorithm achieves energy efficiency by avoiding re-clustering, as the cluster formed is maintained until the end of the event. Clusters are formed by exchanging control messages, thus generating a control overhead. In dynamic networks, such as CRSN, the exchange of control messages causes a delay. Therefore, ESAC suffers from delay in cluster formation.

The energy-aware routing protocol (EAP) referred to in [4] aims at preserving the node's energy by forming clusters. All nodes within a cluster forward the sensed data to CH whose role is to aggregate the collected data and forward them to the BS. As CH consumes more energy than other nodes due to receiving the sensed information, aggregating data and forwarding them to the BS, the role of CH is assumed, on a rotational basis, by all member nodes. EAP minimizes energy consumption by forming clusters with cluster members joining the nearest CH.

The equalized cluster head election routing protocol (ECHERP) is proposed in [5]. It achieves energy efficiency through balanced clustering of sensor nodes. Using the Gaussian elimination algorithm, ECHERP identifies the total amount of energy consumed and thereby selects CHs needed for cluster formation, so that the overall network lifetime is increased. The node which minimizes the total energy consumption within the cluster is elected as CH in

each CH selection round. TDMA slots are created by BS and are broadcast to all nodes by which data packets are transferred between the sensor nodes.

A similar energy-aware routing algorithm (ERA), which is a cluster-based routing protocol, is proposed in [3]. In this protocol, CH is selected based on the residual energy of nodes and on intra-cluster distance. Since CHs have multiple tasks to perform, their energy may be depleted rather easily. Also, if the sensor nodes are not deployed properly in the network, then the distance between CHs may vary, resulting in an increase of intra-cluster distance. This results in quicker energy depletion for a CH. The algorithm constructs a directed graph of CHs as a virtual backbone to aid data routing. That is how ERA balances energy consumption between its CHs.

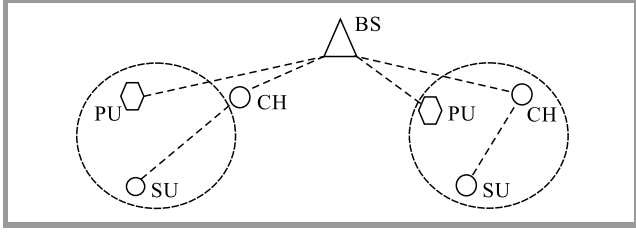
## 3. Implementation Details

In this section, a novel approach to CRSN that emphasizes energy conservation and forms clusters by assigning idle channels to SUs using binomial distribution is described. Sensor nodes send messages over the available channels. In CRSN, an additional constraint of detecting PU activity is to be considered, as CR nodes may utilize the spectrum only in the absence of PUs. So, based on the results of spectrum sensing and on the availability of channels, CR nodes communicate with each other. The proposed protocol forms clusters setting up communication between the cluster member (CM) and CH, and then CH connects with the neighbor CH, until the BS is reached. The communication process may be divided into two categories: between the clusters (inter-cluster) and within the clusters (intra-cluster). During inter-cluster communication, CH collects and forwards the information to BS through the channels available, which are shared with upstream neighboring clusters. During intra-cluster communication, the sensed information is sent to CH using a local channel.

### 3.1. Network Model

Let us consider a CR sensor network  $P = \{S, N\}$ , where  $S = \{s_1, \dots, s_n\}$  indicates the total number of sensor nodes in the network and  $N = \{n_1, \dots, n_{ij}\}$ ,  $i \neq j$  indicates connections created between other peer nodes from  $N$ .  $n_{ij}$  indicates the direct connection between two acquaintance nodes, known as  $n_i$  and  $n_j$ , within the definite time interval  $t$ . All sensor nodes are positioned randomly throughout the grid which examines the updated channels for efficient connection. The network is made up of multiple PUs, SUs with sensing ability, and a centralized BS – as shown in Fig. 1.

Each node maintains a neighbor connectivity table, containing a list of immediate neighbor nodes which are one hop away (Table 1). If the value for node  $j$  is set to 1, then the node becomes an immediate neighbor of node  $i$ . Sensor nodes with the channel sensing ability are deployed. Sensor nodes are aware of the available channel spectrum,



**Fig. 1.** Secondary user communication via primary channel and CH.

since each sensor node supports cognitive analytical features. This makes the sensor nodes aware of which part of the spectrum is currently underutilized. Each node maintains the transport state timer to update channel state information and channel state probability. The SUs identify channel occupancy and availability during the sensing process according to Table 1.

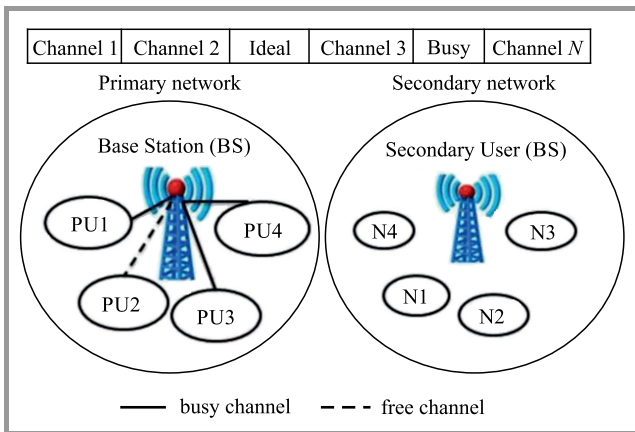
Table 1

Neighbor connectivity and channel occupancy scheme

| Node | A | B | C | D | E | F | G | H | I | Neighbors  |
|------|---|---|---|---|---|---|---|---|---|------------|
| A    | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | B, G       |
| B    | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | A, C, H    |
| C    | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | B, D, F    |
| D    | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | C, G, I    |
| E    | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | F, H       |
| F    | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | C, E, I    |
| G    | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | A, D, H    |
| H    | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | B, E, G, I |
| I    | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | D, F, H    |

### 3.2. Channel Allocation Model

The BS requests the sensors to perform channel sensing in order to identify the free channels. The channel list is maintained at BS and is always kept up to date. Each SU performs local sensing to identify its own vacant channels. Since CRSN is a dynamic environment where PU activity is



**Fig. 2.** Distribution of idle channels for secondary users.

random and PUs may occupy a given channel at any time, channel availability information must be available to the neighboring nodes as well. After performing local sensing, SUs exchange information with their neighbors.

The base station (BS) periodically sends an announcement as a network-wide broadcast message. When all sensor nodes receive it, SUs get initialized with the timer in order to periodically update location- and channel occupancy-related values. All sensor nodes forward their location information to BS which aggregates the collected information and then starts the clustering process. Next, the probability of the node to be a member of a cluster is computed as the ratio between the product of the total number of channels and the collection of idle channels available at the node, and the entire collection of idle channels at all nodes. We use binomial distribution to distribute idle channels sensed by the node, as illustrated in Fig. 2. The grouping and channel allocation process is improved by considering channel overlapping and channel interference computations.

### 3.3. Distance between Nodes and Channel Overlapping

The computation of neighbor distance is based on the position of network region points  $(X, Y)$ . Let node  $i$  and node  $j$  have the position of  $(X_1, Y_1)$  and  $(X_2, Y_2)$ , respectively. Distance  $D_i$  between node  $i$  and node  $j$  is:

$$D_i = \sqrt{|X_1 - X_2|^2 + |Y_1 - Y_2|^2}. \quad (1)$$

The proposed protocol deals with channel overlapping if two or more SUs share the same channel. To cope with this, ICBCA checks for all available channels in the communication. Based on the derived optimal number of CHs, clustering probability for each node is determined by comparing the result with the total number of cognitive sensor nodes in the network. Channel IDs are updated continuously from the entire network area and the list of channels ( $C_L$ ) is framed at each sensor node. Let  $O_L$  indicate the count of overlapping channels. We then check  $O_L$  states from  $C_L$  and update the count. If  $O_L$  is found in  $C_L$  based on network region  $N_R$ , the overlapping counts  $\pm O_{LC}$  are added and  $O_L$  is updated:

$$O_L = \frac{D_{ij} + D_{ti}}{N_R}. \quad (2)$$

Then, the final  $O_L$  is:

$$O_L = \frac{O_L}{O_{LC}}. \quad (3)$$

The channel overlapping state is shown in Fig. 3 and Table 2. If a sensor node falls under the communication range between two clusters, it becomes a one-hop neighbor of two equivalent CHs. This node is referred to as a midway node. Its CH is selected by considering any of the midway sensor nodes as the connection sensor required to reach the next cluster. We refer to the connection sensor node as a one-hop access node. In the other case, if two clusters are non-overlapping but there are sensors falling

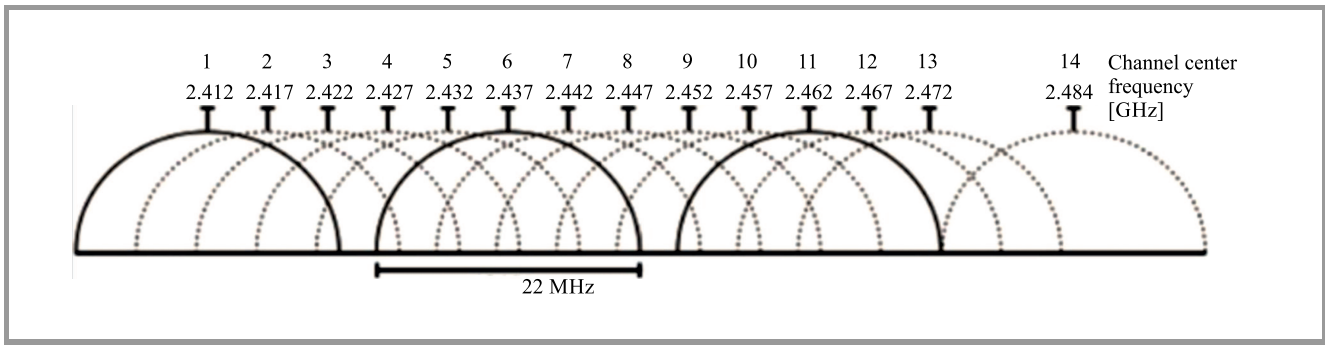


Fig. 3. Channel overlapping illustration.

Table 2  
Channel interference and overlapping

| Node | Channel |   |   |   | Node – channel | Node | Co-channel interference | Overlapping |
|------|---------|---|---|---|----------------|------|-------------------------|-------------|
|      | 1       | 2 | 3 | 4 |                |      |                         |             |
| A    | 0       | 1 | 0 | 0 | A 2            | A    | –                       | B–G         |
| B    | 0       | 0 | 0 | 1 | B 4            | B    | H                       | A–C         |
| C    | 0       | 0 | 1 | 0 | C 3            | C    | –                       | B–D–F       |
| D    | 1       | 0 | 0 | 0 | D 1            | D    | –                       | C–G–I       |
| E    | 0       | 1 | 0 | 0 | E 2            | E    | –                       | F–H         |
| F    | 0       | 0 | 0 | 1 | F 4            | F    | –                       | C–E–I       |
| G    | 0       | 0 | 1 | 0 | G 3            | G    | H                       | A–D         |
| H    | 0       | 0 | 1 | 0 | H 3            | H    | B–G                     | E–I         |
| I    | 0       | 1 | 0 | 0 | I 2            | I    | –                       | D–F–H       |

within communication range of two clusters which can listen to each other, then these sensors intersect two groups. Those sensors are called two-hop connection sensors.

### 3.4. Channel Interference

In CRSN, since PU has the highest priority, the active PUs can use all available channels, causing potential interference. Channel interference is considered to be the region where two or more sensor nodes use the same frequency. Channel interference in a specific cluster may be avoided effectively by adopting a policy under which no currently reachable channel within a cluster is selected, as shown in Table 2. By using the binary co-channel conflict list, we divide channel  $x$  by sharing it with both  $i$ -th and  $j$ -th SUs. Table 3 shows the other channel parameters used in the simulation.

Table 3  
Channel parameters used in simulations

| Parameter            | Value  |
|----------------------|--------|
| Height of antenna    | 1.5 m  |
| Packet size          | 2 MB   |
| Receiver noise level | –2 dBm |
| Antenna gain         | 20 dBi |
| Path loss            | 4 dBm  |

### 3.5. Binary Matrix Formation

Next, the spectrum is divided into several channels, as shown in Fig. 4. The real channel refers to the original spectrum which is available for communication. It divides the frequency band into sub-channels, with the necessary spacing between them. As some channels may be unavailable for use, overlapping may occur, but initially all nodes find a free channel which is represented as the binary matrix in Table 4.

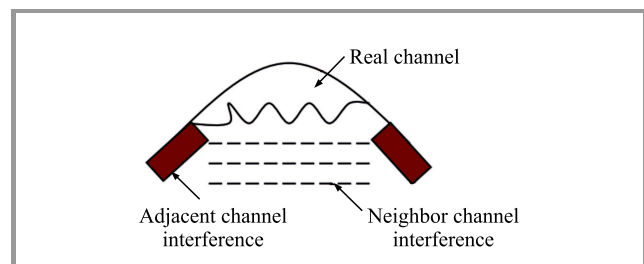


Fig. 4. Sectioning the spectrum into sub-channels.

If the channel is accessible, then the value is set to 1. Otherwise, it is set to 0.

For each potential channel, the incentive cost is computed based on its availability, by checking the distance to PU, throughput and bandwidth. The bandwidth of the communication link determines the packet data arrival rate. The

Table 4  
Binary matrix design

| Node | Channel 1 | Channel 2 | Channel 3 | Channel 4 |
|------|-----------|-----------|-----------|-----------|
| 1    | 0         | 0         | 1         | 0         |
| 2    | 0         | 1         | 0         | 0         |
| 3    | 1         | 0         | 0         | 0         |
| 4    | 0         | 0         | 1         | 0         |
| 5    | 0         | 1         | 0         | 0         |

higher the bandwidth value, the higher the data rate, as less time is required for packets to reach their destination. Based on the connection between two sensor nodes, channel bandwidth may be estimated as:

$$C_B = \text{Payload} \cdot \frac{B_W}{C_T}, \quad (4)$$

where  $C_T$  and  $B_W$  define the current time and bandwidth. Throughput  $T_P$  is defined as the total number of receiving bytes (payload) per second at the receiver end:

$$T_P = \frac{\text{Payload}}{C_T}. \quad (5)$$

If the channel overlapping count is  $> 0$  and the channel interference range is  $> 0$ , then the communication state is computed. The occupied number of channels is updated from the channel list  $C_L$ . We then find the number of sensor nodes  $S$  from the network and update the optimal cluster count  $K$ . Next, channel information  $C_I$  is found based on the number of channels  $m$  and occupied channel list  $O_C$ .

$$C_I = m - O_C. \quad (6)$$

Then, the state of the cluster and CH after each interval  $R$  of the communication range  $C_R$  is computed:

$$C_R = \frac{K}{S - K} \frac{S}{K} \cdot R [\%]. \quad (7)$$

The  $C_I$  value is updated periodically by using channel information from current scenarios  $C_{L-}$ :

$$C_{L-}(m - C_I + 1). \quad (8)$$

The updated channels using the channel value  $\alpha$  are:

$$\alpha = \frac{m}{C}, \quad C_R = \frac{K\alpha C_I}{m, 1}. \quad (9)$$

If the number of updated channels remains unchanged, then:

$$C_{R-} = \min \frac{KC_I}{\sum C}. \quad (10)$$

The effective channel selection is based on transmission range  $C_{R-}$ . This effective channel selection is required to identify the optimal number of available idle channels within the PU coverage area. Next, the probability of a node to become a CH is estimated. The number of sensor nodes

within the PU communication range, known as PU-affected area, with the coverage probability and the number of sensor nodes not covered within PU, is computed from the affected area with the non-coverage probability. These factors are used to evaluate the final probability of the node to become the CH, and then to identify the transmission channel TCcount and the receiving channel RXcount. During communication, static channel probability  $C_P$  for two sensors is:

$$s1 = sC_P \text{ and } s2 = s(1 - C_P). \quad (11)$$

If  $C_P > 0$  and  $C_P < 1$ , channel selection  $C_S$  is:

$$C_S = \min \frac{KC_I}{[(s1m + s2m), 1]}. \quad (12)$$

Further, an available channel  $C_A$  is defined as 1 if the channel is vacant and 0 otherwise. The channels are chosen from Table 2, as available channel  $C_A$  and the value of  $(C_A C_S)$  are computed. In the event of channel overlapping and when the interference range is greater than 0, the channel incentive cost is derived and assigned to each sensor node using:

$$\text{cost} = \frac{C_A \cdot C_S}{(O_I + C_I)}. \quad (13)$$

### 3.6. Avoiding Channel Interferences

Interference between channels occurs when two sensor nodes use the same channel at the same time. Here, we avoid co-channel interference by restricting channel reuse within a single cluster. Channels are assigned to clusters ensuring that the same channel cannot be reused during the specific iteration. Channel nomination is performed based on priority-based allocation. Based on current channel ownership and potential of interference with the neighboring node's channels, the process of clustering sensor nodes is performed. The highest clustering probability of the node based on channel ownership and the lowest channel interference value is estimated. The clustering of sensor nodes is performed among all cognitive sensor nodes in the network. Sensor nodes collect and update information about their neighbors to determine the optimum quantity of clusters needed to cover the entire network area. Then, node density is evaluated as the total number of nodes per unit area. The average cluster distance is estimated by calculating the range from the present CH to all other cluster members in the cluster by checking whether a node is a CH or not.

The largest distance value is measured by taking the highest range value of the present CH to the corresponding cluster group members, as shown in Fig. 5. From these values, we estimate the average power needed to perform data transmission, and cost  $D_I$ .

The inter-cluster communication power rate is computed as  $I_N = kC_O R_P 2D_I$  and intra-cluster communication power rate is computed as  $I_G = 2C_O R_P \sum D_I$ , where  $k = \frac{S}{D^2 D_I}$ ,  $D = \frac{S}{N_S}$ , and  $N_S$  denotes the network size. The total communica-

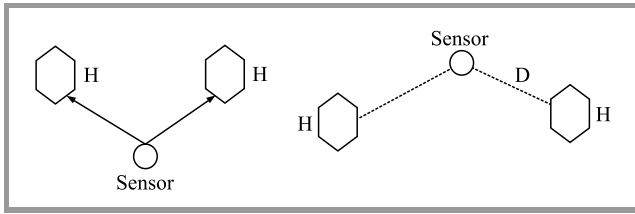


Fig. 5. Sensor and CH communication.

tion power necessary to perform data transmission in inter-cluster  $I_N$  and intra-cluster  $I_G$  is:

$$\text{Total power} = 2C_O R_P E_E + 2E_E K C_O R_P 2D_t . \quad (14)$$

### 3.7. CH Election and Clustering Process

Once the optimal collection of tentative clusters is identified, the lowest power expected to complete the communication process is computed. Then, the probability estimation-based stable CH is selected by using bandwidth, throughput, channel value and distance to SU. Based on this, the node executes the CH decision process to join a particular cluster. The node with the highest probability value becomes the tentative CH.

The selected CH nodes activate the final CH timer to receive a join message from the group members. Upon hearing the tentative CH announcement, non-CH nodes store the ID, location and the probability value of the CH. The non-CH nodes start the wait timer to receive an announcement message from all CH nodes within their coverage. Once the wait time expires, non-CH nodes select the best available CH with the highest probability and lowest distance. The CH nodes keep the member list by accepting the join information from all the members. The CH validates the member list for the joined nodes. If the non-CH node do not receive the final CH announcement message, then it tries to join other potential groups. If there are no available CH within range, the node itself becomes a CH. The node with the highest channel availability rate and the lowest level of interference will become the CH. In certain iterations, the selected nodes should not be reassigned with the same channel until all channels are reassigned based on the round-robin method.

### 3.8. Selection of Fair Channel and Slot Computation

The fairness index is computed by calculating the number of channel allocations assigned to each sensor node during the current iteration. The fair value is calculated by applying the fairness index with the maximum incentive cost and channel communication parameters. For each node, the required data rate is computed by checking all incoming and outgoing traffic. By comparing the required data rate of each node, ICBCA estimates the priority of the data transmission in the node.

During the channel allocation process for each cluster, the channel with the non-overlapping and the minimum interference gets a high fairness index value and a high

data priority rate. The same process is repeated for all formed clusters and then data transmission takes place in the allocated channel. Upon completion of cluster formation, the CH node generates TDMA slots for all its cluster members. The generated TDMA schedules are broadcasted to all member nodes within the cluster. Based on the received TDMA slot, the SUs perform the data transmission operation.

## 4. Performance Evaluation

In this paper, Network Simulator NS-2 was used to simulate the proposed ICBCA protocol and to compare the results with existing clustering algorithms, i.e. DSAC [11] and CogLEACH [12].

Figures 6 and 7 show the comparison of protocols in terms of energy consumed by the node and in terms of the node's residual energy. CogLEACH considers the probability of finding idle channels and the node which has more idle

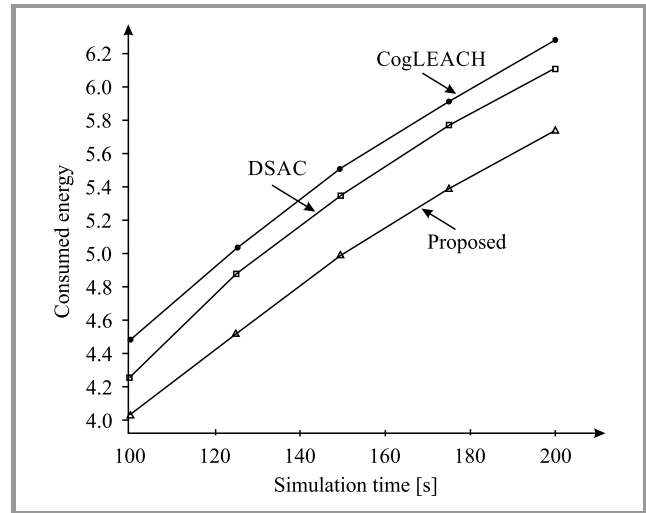


Fig. 6. Simulation time vs. average energy consumed by the node.

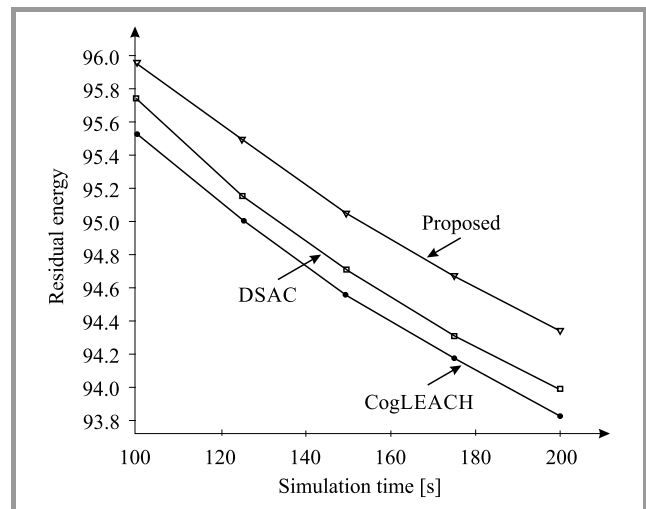


Fig. 7. Simulation time vs. average residual energy of the node.

channels becomes the CH. It prohibits SUs from transferring packets if the entire spectrum is busy, thereby saving energy. However, in a dynamic environment like CRSN, this approach is not suitable. In DSAC, energy is saved based on the CH rotation policy. It considers the optimal number of clusters by avoiding overlapped channels. ICBCA offers better energy efficiency than other solutions, since both channel interference and channel overlapping are taken into consideration. In terms of residual energy, ICBCA has better efficiency too when compared with CogLEACH [12] and DSAC [11].

The throughput of CogLEACH and DSAC is comparatively lower than that of ICBCA, as both of them do not support separate channels for transferring control messages (Fig. 8). Although DSAC relies on channel overlapping and interference to select the best channel, priority and fairness-based channel selection are not used. ICBCA, in turn, uses separate channels for control messages and for data transmission. It also performs priority-based channel selection, which results in a better throughput.

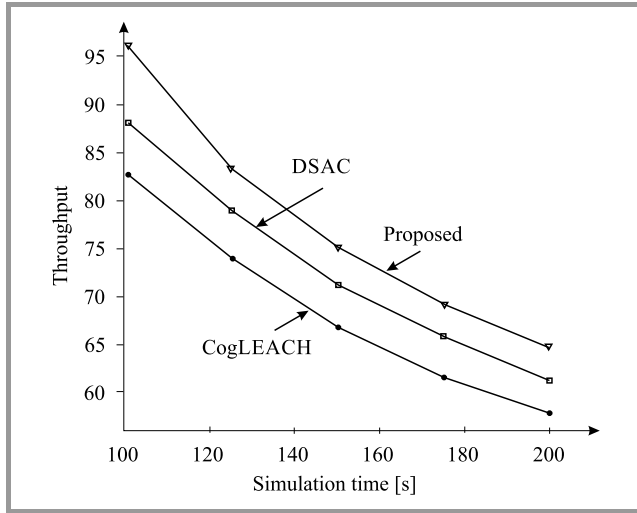


Fig. 8. Simulation time vs. throughput.

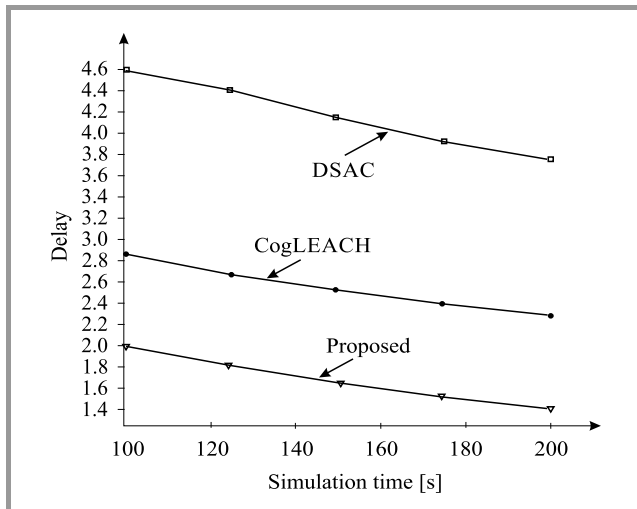


Fig. 9. Simulation time vs. end-to-end packet delay.

Figure 9 compares packet delay between the protocols. As mentioned earlier, ICBCA has a separate channel for sending data packets and for exchanging control messages, due to which the destination nodes suffer from a lower delay. Since ICBCA implements cluster-based channel allocation, allocating a dedicated channel only after the cluster is formed, the proposed work offers a better packet delivery ratio compared with DSAC and CogLEACH (Fig. 10).

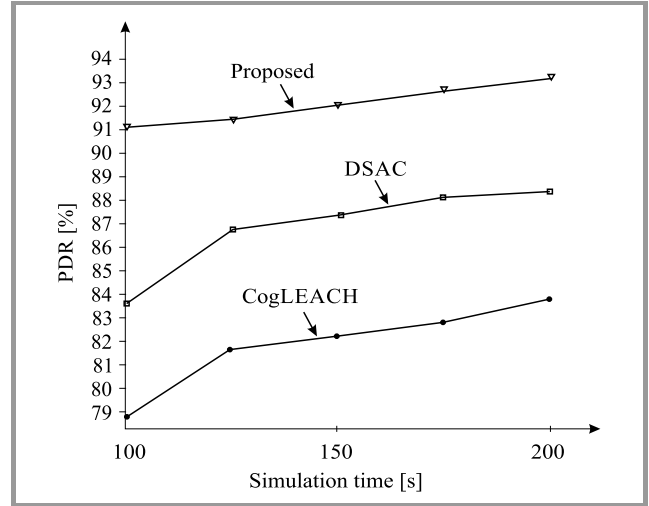


Fig. 10. Simulation time vs. packet delivery ratio.

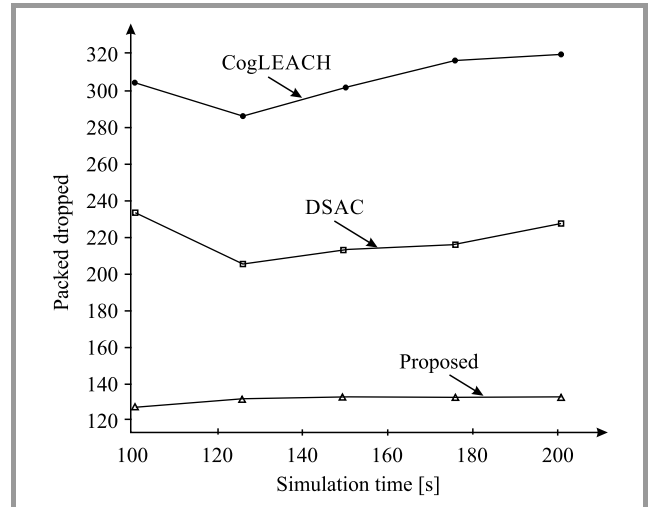


Fig. 11. Simulation time vs. packets dropped.

In ICBCA, packet drop is lower when compared with DSAC and CogLEACH, as both these protocols do not consider priority and fairness-based allocation of the dedicated channel. Figure 11 shows the comparison of the numbers of packets dropped.

## 5. Conclusion

In this paper, a novel clustering algorithm is implemented, relying on fairness- and priority-based allocation of chan-



nets to clusters. According to our knowledge, the existing literature does not describe any research in which a channel assignment scheme considering data rates and quality-of-service (QoS) parameters would be relied upon. In this area, ICBCA shows a significant improvement in the network lifetime of nodes. Simulation results show that ICBCA offers better performance in terms of throughput, energy consumption, packet delay and packet delivery ratio when compared with other existing CRSN clustering algorithms, such as DSAC and CogLEACH.

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# Cooperative Spectrum Sensing in Cognitive Radio Networks: A Survey on Machine Learning-based Methods

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<https://doi.org/10.26636/jtit.2020.137219>

**Abstract**—The continuous growth of demand experienced by wireless networks creates a spectrum availability challenge. Cognitive radio (CR) is a promising solution capable of overcoming spectrum scarcity. It is an intelligent radio technology that may be programmed and dynamically configured to avoid interference and congestion in cognitive radio networks (CRN). Spectrum sensing (SS) is a cognitive radio life cycle task aiming to detect spectrum holes. A number of innovative approaches are devised to monitor the spectrum and to determine when these holes are present. The purpose of this survey is to investigate some of these schemes which are constructed based on machine learning concepts and principles. In addition, this review aims to present a general classification of these machine learning-based schemes.

**Keywords**—cognitive radio, cooperative spectrum sensing, IEEE 802.22, machine learning, spectrum sensing.

## 1. Introduction

Cognitive radio (CR) is an intelligent radio technology capable of determining the frequencies that are in use, detecting the available spectrum holes, and then reconfiguring transceiver parameters based on the radio environment information obtained. Spectrum sensing (SS) is the first task in cognitive radio life cycle. It has been gaining in significance, since it allows to detect spectrum holes.

Several authors have attempted to come up with different classifications of spectrum sensing methodologies utilized in cognitive radio networks (CRNs) [1]–[3]. The authors in [1] categorized SS schemes based on the mode that was relied upon in SS decision making, namely local or cooperative sensing. In [2], [3], the authors categorized SS schemes based on the radio communication types adopted, mainly into narrow- and wide-band sensing. Although cognitive radio is based on artificial intelligence (AI) concepts and principles [4], only a few works were devoted to machine learning schemes relied upon in cooperative spectrum sensing. In contrast, this survey examines different machine

learning-based schemes proposed for cooperative spectrum sensing (ML-based CSS) in CRN.

This paper concludes that three main types of ML-based CSS schemes exist. The first one is known as unsupervised-based CSS. Here, features are extracted using a suitable feature extraction algorithm. These features are then fed into a machine learning (ML) model that tends to divide data into different clusters that are characterized by their corresponding centroids. Consequently, it makes a decision about the channel's availability status by comparing each centroid with a predefined threshold value. This approach offers reasonable performance levels but its accuracy is rather low compared to other methods [5].

The second approach is referred to as supervised-based cooperative spectrum sensing (CSS). In contrast to the previous method, specific features are fed to a classifier along with their labels that are assigned to them for training purposes. The training process then aims to find the best decision boundary that is capable of separating this labeled data into classes. Therefore, the decision is taken based on that boundary. This approach is more accurate than others, but the labeling process generates some overhead during the training phase [6]. The third method is known as reinforcement learning-based CSS. Here, the algorithm focuses on a certain problem, like throughput and energy consumption. Then, it tries to solve that problem by finding the optimal CSS policy which returns the highest cumulative reward. However, used in CSS, this approach creates several challenges, such as high computational complexity and the requirement for a policy that is needed to make decisions in real time [7].

The remaining part of this paper is organized as follows: Section 2 presents the background of SS tasks performed in CRNs. The general model of CSS, as well as the classification of ML-based CSS schemes, are presented in Section 3. In Section 4, feature extraction methods are introduced, while ML-based CSS schemes are explained in Section 5. Section 6 discusses the problem of evaluating performance of ML-based CSS schemes and, finally, Section 7 contains some concluding remarks.

## 2. Spectrum Sensing Task in CRNs

### 2.1. Cognitive Radio Technology: An Overview

CR is defined as intelligent radio being aware of the surrounding environment and knowing the frequencies that are in use. One of the most notable features of cognitive radio networks (CRN) is their ability to switch between different radio access methods, as well as ability to transmit within different portions of the radio spectrum [8]. CR is the primary component of any CRN structure and there are two main types of users operating CRNs. They are classified into [9] two categories.

Primary users (PUs) are licensed users who have a legal right to use a part of the spectrum. However, PUs are not granted the exclusive use of that part of the spectrum, they are merely granted a higher priority than other users, and enjoy additional interference protection guarantees.

Secondary users (SUs) are the remaining users who have the opportunity to use a part of the spectrum alongside the PUs. However, their right is conditioned on the activities of the PUs and they can use their part of the spectrum if it is temporarily not occupied by the PUs, or if they are able to share the spectrum. Small scale CRNs consist of one PU and a few SUs [5], [6], [10]–[16] with each one being able to use multiple antennas [12]. In contrast, large scale CRNs consist of more than one PU [4], [7], [17]–[21]. However, in modeling CRNs, multiple PUs may operate within the same CRN area and each one may occupy several sub-bands [7], [21].

### 2.2. Spectrum Sensing

Spectrum sensing (SS) is a task that is of key importance in CR. CR learns and is conscious of the surrounding environment, trying to detect any existing spectrum holes. SS was first formulated as a binary hypothesis test for radar signal detection [22]. Later, the same test was used to describe the general SS problem:

$$y(k) = \begin{cases} w(k), & H_0 \\ \alpha s(k) + w(k), & H_1 \end{cases}, \quad (1)$$

where  $y(k)$  is the received signal that was received by SU under ambient noise  $w(k)$  (usually  $w(k)$  is the additive white Gaussian noise (AWGN) with 0 mean and variance of 1),  $s(k)$  is the PU's signal and  $k = 1, \dots, K$  are signal samples received by the SU. Parameter  $\alpha$  denotes the channel's fading coefficient between the PU and the SU. The SU checks whether PU's signals are present and the channel is considered **idle** under the null hypothesis ( $H_0$  condition) and **busy** under the alternative hypothesis ( $H_1$  condition). Two types of errors are faced in connection with detection of the PU's signals. The first one, type 1, is a false alarm (FA). The decision made indicates that the PU exists, while it is not present, in fact (the decision is  $H_1|H_0$ ). The other error, type 2, is known as missed detection (MD) and occurs when a busy channel is identified as being idle (the decision

is  $H_0|H_1$ ). The two types of errors referred to above are described by the probability of a false alarm ( $P_{FA}$ ) and of a missed detection ( $P_{MD}$ ), respectively. The IEEE 802.22 work group recommended that  $P_{FA}$  should not exceed 10%, while the probability of detection  $P_D = 1 - P_{FA}$  being higher than 90% [23]. If type 1 errors occur frequently, i.e.  $P_{FA}$  increases, the SU will lose numerous opportunities to use the communication channel concerned. In contrast, if type 2 errors occur frequently, i.e.  $P_{MD}$  increases, then interference between PU and SU is inevitable [22].

The task of SS may be performed in non-cooperative or cooperative modes. In the non-cooperative mode, each SU determines the channel's state individually. This mode is suitable when the nodes are unable to share their SS information. In cooperative spectrum sensing (CSS), SUs work together to determine the channel's status. CSS offers a high accuracy level, as all SUs take part in making a shared decision about the channel's condition.

## 3. ML-based Cooperative Spectrum Sensing

### 3.1. General Model of ML-based CSS

In conventional CSS schemes, an SU investigates whether the PU's signal exists and makes a decision about the channel's availability status. Such an approach to CSS is based on ML schemes (ML-based CSS) and encompasses several steps that are shown in Fig. 1. After aggregating the sensing information from all SUs, useful features that differentiate the PUs' signal samples are extracted. Then, these features are fed to the ML model used for decision making. Lastly, the final sensing decision is made upon the result obtained with the use of the ML model. Several ML algorithms relied upon in numerous research projects will be explained next.

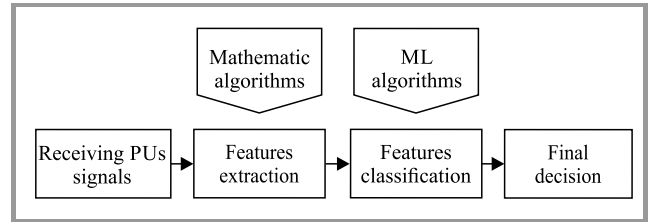


Fig. 1. General model of ML-based CSS schemes.

### 3.2. Classification of ML-based CSS

ML-based CSS schemes, as shown in Fig. 2, are generally classified based on the ML algorithm used. The three main types are unsupervised ML-based CSS, supervised ML-based CSS, and reinforcement ML-based CSS. In unsupervised ML-based CSS, the features are fed to the model without any labels during the training phase [10]–[13], [15], [17], [19], [21], [24], [25]. Many unsupervised machine learning algorithms may be found in the literature, some of which are mentioned in Fig. 2. Clustering paradigms are

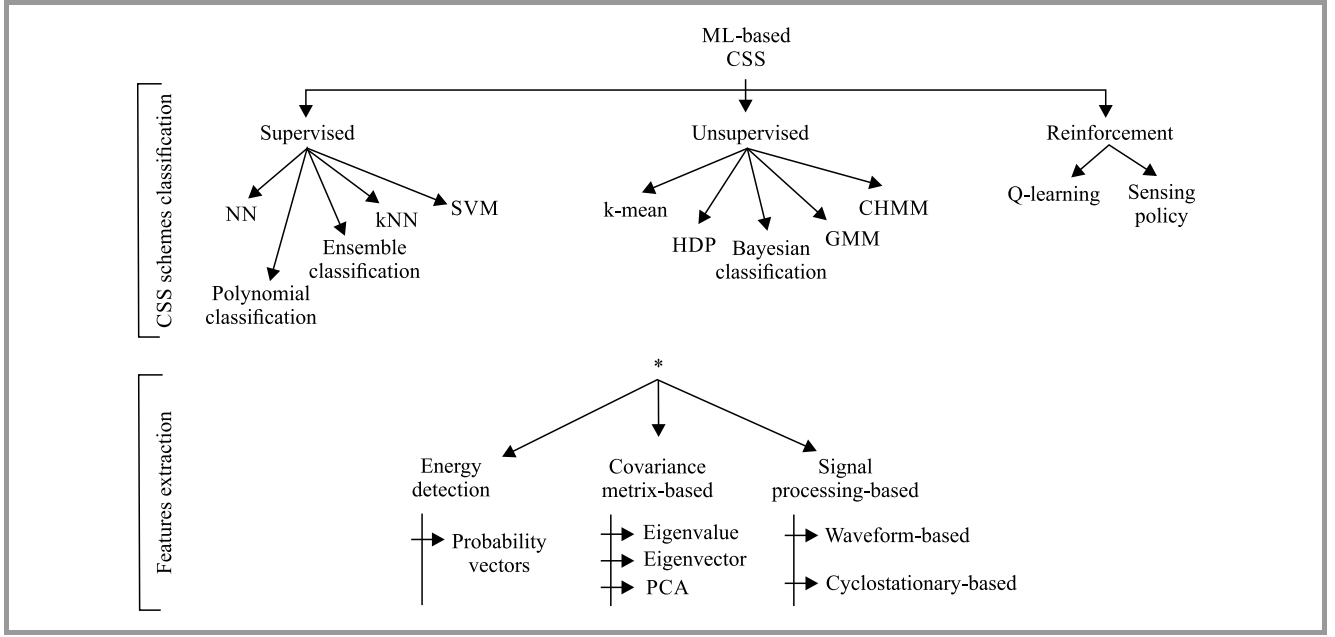


Fig. 2. Classification of ML-based CSS.

the most famous ones, e.g. the very well-known k-means clustering algorithm that will be discussed later in this paper (alongside its variant, the Gaussian mixture model (GMM)).

On the contrary, the training of supervised ML-based methods requires that features be fed to the classifier along with their labels [4]–[6], [14], [16], [20], [26]–[28]. Artificial neural networks (ANN) are the most famous supervised machine learning model. They come in two variants, with traditional architecture and deep neural networks architecture that serves as a basis for modern deep learning schemes. Other examples of the supervised model include support vector machines (SVM), k-nearest neighbors (KNN), and ensemble classifiers. This paper will focus primarily on supervised-based CSS contributions that utilize both neural networks and support vector machine architectures, since they are the most famous and the most commonly used architectures.

Reinforcement ML-based methods consider certain characteristics of the signal in SS [7], [18] such as power consumption, throughput, energy efficiency, etc. The agent is provided with rewards in order to evolve its behavior. These rewards are constructed depending on which problems need to be solved. Q-learning is a famous technique for reinforcing learning algorithms.

Regardless of which ML algorithms (k-mean, GMM, SVM, ANN) are used, several mathematical algorithms have been innovated in order to extract useful features from the signal. As shown in the bottom part of Fig. 2, these algorithms are divided into three categories. The first category covers energy detection-based (ED) algorithms [4]–[6], [10], [14], [16], [17], [20], [21], [26], [27]. Here, energy samples of the PU's signal are collected and are then transformed into energy vectors. The second category covers signal processing-based solutions [14], [16],

[19], [25]. Here, the focus is placed on finding circularly characteristics that help differentiate between transmitted and noise signals. The third category is known as the covariance matrix [11]–[13], [15], [24] and relies on constructing feature matrices from the sensing samples, and on performing specific computations. These methods are presented in detail in Section 4.

## 4. Feature Extraction Methods

Feature extraction is the second phase of the general ML-based CSS model. Various algorithms have been designed in order to perform the feature extraction task. This section will explain some of these algorithms.

### 4.1. ED-based Feature Extraction

A general CSS model encompasses  $P$  PUs and  $N$  SUs. In numerous research projects, a single PU is used, since it reduces the degree of complexity. This paper will also focus on a single PU (unless mentioned otherwise) in order to facilitate the explanation. When the  $n$ -th SU receives  $K$  samples of the transmitted PU's signal, it computes the average energy ( $E$ ) level of the signal:

$$E_n = \frac{1}{K} \sum_{k=1}^K |Z_n(k)|^2, \quad (2)$$

where  $|Z_n(k)|^2$  is the energy value corresponding to  $k$ -th sample and estimated by SU  $n$ . After computing  $E$  value by all  $N$  SUs, the column vector of the energy values  $\{E_1, E_2, \dots, E_n, \dots, E_N\}^T$  is fed to the ML model as a feature vector to make the final sensing decision. **Probability vectors method** is one of ED-based feature extraction scenarios. This method reduces multidimensional energy vec-

tors into two-dimensional probability vectors [11]. In this method, new probability vectors of the two binary hypotheses ( $H_0$  and  $H_1$ ) are defined under the probability density function (PDF) of the multivariate Gaussian distribution, taking the mean and the covariance matrix of each hypothesis as input parameters and constructing the probability vector based thereon.

#### 4.2. Covariance Matrix-based Feature Extraction

The  $S_i$  sensing matrix is constructed based on the PU's signal received. Thanks to the random matrix theory, covariance matrix  $R$  is reconstructed from that sensing matrix:

$$R = \frac{1}{K} \sum_{k=1}^K S_i \cdot S_i^T. \quad (3)$$

Several proposals stem from this approach, including **eigenvalue/eigenvector methods**. Let us suppose that  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n \geq \dots \geq \lambda_N$  are the eigenvalues of covariance matrix  $R$ . Then, the eigenvector ( $\vec{v}_n$ ) of  $R$  corresponding to the eigenvalue may be computed as:

$$R\vec{v} = \lambda \vec{v}. \quad (4)$$

Numerous schemes relied on the eigenvalue/eigenvector in several ways, as a feature of the ML model. For example, the ratio of the maximum and minimum eigenvalues (MME) and the difference between the maximum eigenvalue and the average eigenvalue (MSE) are used in [11], [24], whereas the improved MME (IMME) and the improved MSE (MSE) are used in [12] after computing the principal component of the eigenvector of matrix  $S_i$ .

#### 4.3. Signal Processing-based Feature Extraction

Signal processing is based on transforming the received PU's signal from the time domain to the frequency domain, using the Fourier transform. **Waveform-based detection (WFD)** or **coherent-based detection (CD)** and **cyclostationarity-based feature detection (CFD)** are the most common techniques of this type. CD assumes that patterns of the control signal, such as the pilot, preambles, spreading sequence, midambles, etc. are known and can be efficiently utilized [14]. The notion of cyclostationarity, as used in CFD, indicates the periodic characteristic of a certain signal. Usually, cyclostationarity of the received PU signal is described in terms of its mean and auto-correlation [29], [14]. In contrast, noise signal does not have such periodic characteristics, so the difference between the transmitted signals and noise may be relied upon. The presence of a cycle-stationary pattern of the PU signal received may be determined using the so-called spectral correlation density function (SCD) [14]. In order to determine whether a PU signal is present, one may rewrite Eq. (1) using the SCD function, as:

$$S_{y(k)}^\omega[k] = \begin{cases} S_{w(k)}^\omega[k], & H_0 \\ |\alpha|^2 S_{s(k)}^\omega[k] + S_{w(k)}^\omega[k], & H_1 \end{cases}, \quad (5)$$

where  $S_{y(k)}^\omega[k]$  is the SCD of the transmitted PU signal at some cyclic frequency  $\omega$  and  $\alpha$  is the channel gain coefficient.

## 5. ML-based Cooperative Spectrum Sensing Schemes

A large number of researchers considered applying ML algorithms in constructing CSS systems. Table 1 summarizes some of these algorithms and the feature extraction methods used. The classification of these research projects was explained in Subsection 3.2, where they are categorized into supervised, unsupervised and reinforcement ML-based approaches. This section explains how the problem of CSS may be tackled using these algorithms.

### 5.1. Unsupervised-based CSS

After collecting a sufficiently large number of training feature vectors created by any type of feature extraction algorithm, the ML model is trained to make a decision about the channel's availability state. In unsupervised methods, these features are fed to the ML model to produce the global decision about PUs' state. Several released proposals [21], [15], [25] suggested applying different types of unsupervised learning algorithms to resolve the CSS problem in CRNs. This part explores some of these most prominent algorithms.

K-means is an algorithm that maps a collection of data samples into non-overlapping clusters [15]. The assignment of a certain point to a given cluster is determined based on a distance measure (usually Euclidean distance). The point will be assigned to the cluster whose center is closest to that point. Let us assume that the collection of feature vectors constructs a set called  $\Psi$ . Each  $\Psi$  represents a cluster and is indexed by  $j$  ( $j \in \{0, 1\}$  under the binary hypotheses condition). Each cluster has its own centroid  $C_j$  representing the arithmetic mean of that cluster. Therefore, the distortion function  $\Theta$  computes, for all samples, the overall square distance to the corresponding centroid. Here, the k-means algorithm aims at minimizing the distortion function  $\Theta$ .

After training the k-means model, the members and the centroid of each cluster are determined. In the testing phase, the model is able to predict the suitable decision, i.e. the channel's availability state. Let two clusters  $i$  and  $j$  represent the channel's availability and unavailability states, respectively.  $l'$  denotes a new test vector (data point) that needs to be assigned to a cluster. Then, the model decides about the membership based on a predefined threshold  $\zeta$ :

$$\frac{\|l' - C_i\|}{\|l' - C_j\|} \leq \zeta. \quad (6)$$

This means that vector  $l'$  is classified into the cluster  $C_i$  (i.e. the channel is available) if the ratio of the distance between that vector and  $C_i$  and the distance between that vector and  $C_j$  is lower than threshold  $\zeta$ .

Table 1  
Classification table

| Reference | ML algorithms types | ML algorithms, number of clusters                                           | Features extraction                                        |
|-----------|---------------------|-----------------------------------------------------------------------------|------------------------------------------------------------|
| [10]      | Unsupervised        | k-means, $k = 2$                                                            | ED                                                         |
| [11]      | Unsupervised        | GMM and k-mean, $k = 2$                                                     | Eigenvalue/eigenvector                                     |
| [12]      | Unsupervised        | k-medoids, $k = 2$                                                          | IMSE/IMME                                                  |
| [13]      | Unsupervised        | Fuzzy c-means, $c = 2$                                                      | Geodesic distance                                          |
| [17]      | Unsupervised        | Kernel fuzzy c-means, $c = 3$                                               | Emax                                                       |
| [24]      | Unsupervised        | WEMD (with k-means), $k = 2$                                                | DMEAE/DMM                                                  |
| [19]      | Unsupervised        | HDP, cls = 9, 18, 36                                                        | Fourier coefficients (CFD)                                 |
| [21]      | Unsupervised        | Bayesian learning model (BP-SHMM)                                           | ED                                                         |
| [15]      | Unsupervised        | k-means (multi-bands based), $k = 2$                                        | Eigenvalue                                                 |
| [25]      | Unsupervised        | Blind-CHMM (k-means-based)                                                  | CWT (singularity detection)                                |
| [5]       | Supervised          | Lin/poly-SVM, cls = 2                                                       | Probability vectors                                        |
| [4]       | Supervised          | 2-phase SVM, cls = 2                                                        | ED                                                         |
| [6]       | Supervised          | DNN (CNN)<br>ED (as inputs)                                                 |                                                            |
| [20]      | Supervised          | CSVM (multi-class SVM)                                                      | ED                                                         |
| [14]      | Supervised          | Lin/second-order PC (polynomial classifier)                                 | ED/CFD/CD                                                  |
| [16]      | Supervised          | Ensemble classifier (based on decision trees and AdaBoost algorithm)        | CFD                                                        |
| [26]      | Supervised          | Back propagation neural network<br>ED and LRS-G <sup>2</sup> (as inputs)    |                                                            |
| [28]      | Supervised          | SVM                                                                         | Beamformer-aided                                           |
| [27]      | Supervised          | CSVM (multi-class SVM)                                                      | ED (for the received signal and the residual energy of SU) |
| [18]      | Reinforcement       | Sensing policy (based on $\epsilon$ -greedy policy)<br>Q-value (throughput) |                                                            |
| [7]       | Reinforcement       | POMDP (Q-learning-based)<br>Q-value (number of idle channels)               |                                                            |

The Gaussian mixture model (GMM) provides a smoother membership function than the k-means algorithm. The distribution function of GMM may be defined under the Gaussian density function  $\mathcal{N}(l|\mu_j, R_j)$  which indicates that training vector  $l$  belongs to cluster  $j$  while the mean  $\mu_j$  and the covariance matrix  $R_j$  being its parameters [11]. The training phase in GMM involves estimating parameters  $\mu_j$ ,  $R_j$  and  $\pi_j$  for each class, where  $\pi_j$  is a mixing probability coefficient defining how big/small the Gaussian function will be. The estimation process is performed using an expectation-maximization algorithm, with the details thereof described in [11]. Finally, the testing vectors are assigned to each cluster based on this classifier rule (assuming the binary hypotheses test is adopted):

$$\ln \frac{\pi_1 \mathcal{N}(l'|\mu_1, R_1)}{\pi_2 \mathcal{N}(l'|\mu_2, R_2)} \geq \zeta. \quad (7)$$

Then, the test vector is assigned to the “channel available” cluster when the previous equation is satisfied, and to the “channel unavailable” cluster otherwise.

As stated above and as may be seen in Table 1, unsupervised CSS methods are widely adopted in many research projects. K-means with ED-based feature extraction is adopted in [10], whereas geodesic distance as feature vectors with the fuzzy c-mean is proposed in [13]. In addition, paper [17] proposes filtering the energy vector collected in order to get maximum one vector as a cleaned feature of the kernel fuzzy c-means. The eigenvalue/eigenvector features with k-means were adopted in [12], while they are used with GMM in [11]. Non-parametric Bayesian learning model hierarchical Dirichlet process and beta process sticky hidden Markov model (BP-SHMM) are proposed [19] and [21], respectively. The hierarchical Dirichlet process converts the received signal into the frequency domain in order to find the Fourier coefficients. Then, it uses these coefficients as feature vectors, whereas the beta process sticky hidden Markov model is ED-based.

Additionally, article [24] introduces a signal processing scheme called wavelet empirical mode decomposition (WEMD). This scheme combines the empirical mode decomposition algorithm and the wavelet threshold algo-



rithm in order to remove noise components, hence reducing noise effects. Then, after filtering noise from the transmitted signal, features are extracted using the difference between the maximum eigenvalue and the average energy, and the difference between the maximum and minimum eigenvalue methods. Finally, k-means is adopted as an ML model, relying on these two types of features extraction methods. Last but not least, a blind continuous hidden Markov model (blind-CHMM) scheme is proposed in [25]. This algorithm is capable of recognizing the transmitted power level of the PU. It uses the k-means algorithm to detect the presence of a PU, as well as the continuous wavelet transform method for feature extraction. Moreover, in that scheme, two strategies are proposed. The first applies the ED method to build the observation sequence before computing the continuous wavelet transform, while the other uses the minimum-maximum eigenvalues as feature vectors.

## 5.2. Supervised-based CSS

To train a supervised-based CSS system, the features are extracted from the received signal and are then assigned with a certain label corresponding to their class affiliation. Due to the binary hypothesis, binary labels may be represented in the following manner. Label 1 indicates that PU is absent and the channel is available, whereas label 0 indicates that PU is present and the channel is not available. However, multi-class hypotheses are more complex, thereby resulting in the need to form label codes corresponding to each class [20], [27], [28].

Support vector machines (SVM) and neural networks (NN) are the most famous examples of supervised-based CSS. SVM is an algorithm that aims to find the optimal hyperplane that leaves the maximum margin from all potential classes. Due to the binary hypothesis test, two potential classes reflect the channel's availability state. In more complicated scenarios, the multi-class SVM with one-versus-all (OVA) strategy is used. OVA is a commonly used approach in which a certain class is marked as the positive class, and the remaining classes are marked as negative [20], [27].

Many research contributions proposed using SVM as a machine learning model. Probability vectors were proposed as feature vectors for SVM, with linear and polynomial kernels, in [5], whereas [4] proposed applying SVM in two phases, along with the ED-based method, for feature extraction. In the first phase, the data samples are randomly split into two sets. The first set contains training samples, whereas the other is used for testing (the true labels are omitted). In the second phase, the samples with their labels work as a training set. The reason for applying the second phase is to neutralize the misclassification of the first phase. Finally, the multi-class SVM ED-based approach with multiple PUs is adopted in [20], [27], and the beamformer-based approach is adopted in [28].

Neural networks are a collection of algorithms encompassing a set of interconnected virtual neurons intended to

work in a manner closely resembling a biological neuron network or electronic structures. A few CSS schemes based on neural networks were proposed in the literature. One is deep CSS (DCSS), based on convolutional neural networks (CNNs) from [6].

Generally, a CNN architecture is a layered architecture with each type of layers having a specific task. Three types of layers may be identified: convolutional layers, max pooling layers, and fully connected layers. The general architecture may be mainly divided into two parts: the convolution part and the fully connected part (FC), as shown in Fig. 3. The convolution part contains the convolutional layers and the maximum pooling layers. The convolutional layers contain a set of filters (kernels) that are convolved with the input matrix. The main task of these filters is to extract the most important features of the input matrix. Unlike in the case of manually-defined filters, the contents of these filters are determined during the training of the network. This means that the process of feature extraction becomes a part of the network learning process, and the traditional stage of manual feature engineering disappears. The size of the kernel (e.g.  $3 \times 3$ ) is a hyperparameter that should be decided in advance. Usually, the size of  $3 \times 3$  is a good choice that works very well.

The objective of the max pooling layers is to downsample the input matrix to reduce data dimensionality, leading to lower computational costs. Maximum pooling helps also in reducing overfitting in the network. Max pooling divides the input matrix into distinct blocks and the maximum value from each block is kept only. So, if the input is  $4 \times 4$  matrix and the block size is set to equal  $2 \times 2$ , the output of the max pooling layer will be a  $2 \times 2$  matrix. Rectified linear unit (ReLU) layers are also found in the CNN architecture. The purpose of the ReLU unit is to increase nonlinearity of data.

Fully-connected layers are needed for the classification process. They resemble a traditional feed-forward neural network. The output of the convolution part is flattened into a column vector and fed to this feed-forward neural network for classification purposes.

The model proposed in [6] uses the CNN architecture described above. In this model, a small-scale network with one PU is adopted that may operate several sub-bands. Each SU examines the existence of PU within a given sub-band and produces a two-dimensional array from the labeled sensing data, as illustrated in Fig. 3 (the label assumes the value of 0 or 1, indicating whether the channel is available or not). Then, the 2D array is fed to the convolution part which encompasses the  $3 \times 3$  convolution layer, the rectifier linear unit layer and the max pooling layer. The first layer extracts spatial correlation of the sensing data, whereas the remaining layers deal with the non-linear behavior and reduce the size of the sensing data, respectively. The FC layer multiplies the weights and adds the biases to the result produced by the previous layer. Finally, the softmax operator is used at the end of the FC layer for making decisions regarding the presence of PUs.

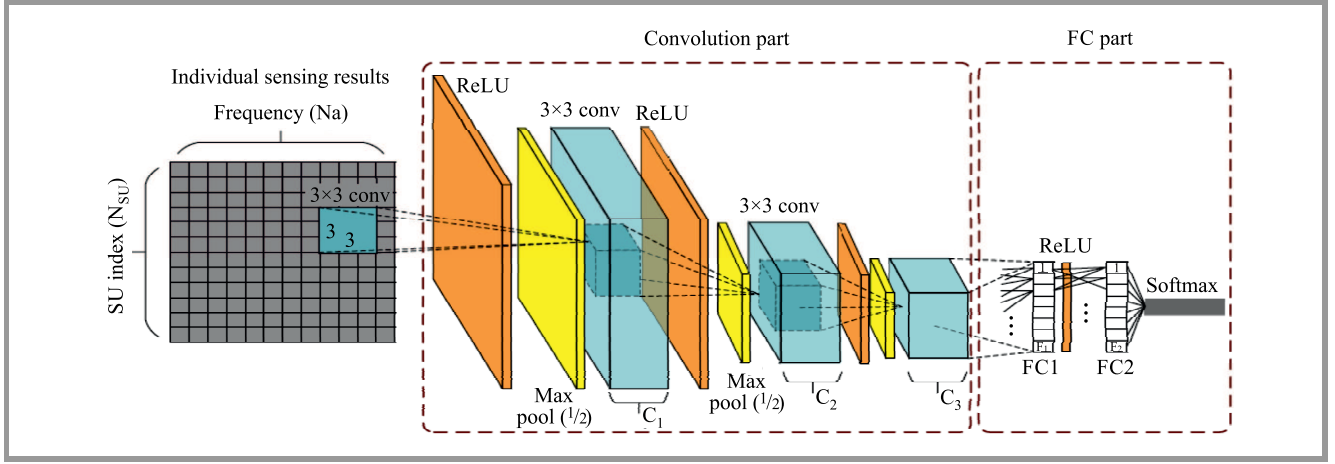


Fig. 3. CNN model for DCSS [6].

Another scheme that is based on traditional (non-deep) artificial neural networks (ANN) was proposed in [26]. This scheme utilizes the ED and Zhang statistic from likelihood statistic test to train the ANN for decision making purposes. Moreover, another supervised scheme, i.e. the ensemble-based classifier, is introduced in [16]. The design of this scheme is based on the decision tree and on AdaBoost algorithms, while the features extraction method that is relied upon is CFD-based.

### 5.3. Reinforcement CCS

Reinforcement learning means that the action is taken which brings the maximum reward.  $Q$ -value  $Q(a)$  denotes the reward granted, corresponding to action  $a$  taken. In a reinforcement ML-based paradigm, the agent initially estimates the action that should be taken next time, based on a certain policy [18], [7]. Then, it gets a reward  $r(a)$  that reflects the validity of the action taken. There is a frequently used term when speaking of reinforcement ML-based algorithms – namely balance between exploration and exploitation. Exploitation indicates that the agent takes an action upon the highest estimated  $Q$ -value, whereas exploration indicates the random selection of a certain action regardless of action-value estimation [18].

Sensing based on the  $\epsilon$ -greedy policy is proposed in [18], whereas an efficient sub-band selection policy based on replicated  $Q$ -learning is proposed in [7]. In the sensing policy, the reward represents the instantaneous throughput for the sub-band selection. Assuming that the sub-band is denoted with  $sd$ , in this policy the SU which had the permission to access the  $sd$  feeds back information about the achieved throughput towards the FC. Herein, FC updates the  $Q$ -value for a particular SU by comparing its decision with the SU's decision. Then, the obtained reward is updated based on the following rule:

$$r_{t+1} \sim \begin{cases} dn_{t+1}(SU, sd), & dn_{t+1}(FC, sd) = 1 \\ Q_t(SU, sd), & dn_{t+1}(FC, sd) = 0 \end{cases}, \quad (8)$$

where  $dn_{t+1}(SU, sd)$  indicates the local decision taken by the SU on  $sd$  while  $dn_{t+1}(FC, sd)$  is the global decision initiated by the FC. After updating all  $Q$ -values based on that rule, FC exploits its knowledge and informs SUs to sense the sub-band that has the maximum  $Q$ -values. Finally, [7] proposed an efficient sub-band selection policy which is modeled based on the partially observable Markov decision process (POMDP) in which the highest rewards are granted to the action that identifies the largest number of idle channels, and the lowest rewards are granted based on the lowest number of idle channels identified.

## 6. Comparison and Discussion

This section analyzes the performance of several ML-based CSS approaches at low signal-to-noise ratio (SNR) levels. The performance of ML-based CSS schemes was measured based on the receiver operating characteristic (ROC) curve, i.e. a plot showing the probability of detection PD versus that of a false alarm PFA. According to the ROC curve, performance of the proposed schemes is somewhat convergent, regardless of which ML type (unsupervised or supervised) is used for decision making. Moreover, considering the IEEE 802.22 standard, one may observe that the majority of schemes offer superior performance even at low SNR levels, as may be seen in Table 2a-b. Performance of the reinforcement-based CSS scheme is usually measured based on the throughput obtained, as shown in Table 2c.

### 6.1. ROC for Performance Evaluation

Unsupervised-based CSS schemes showed good performance – according to the ROC curve presented in Table 2a. The unsupervised wavelet empirical mode decomposition scheme [24], with the difference between maximum eigenvalue and the average energy, and the difference between maximum and minimum eigenvalue features, showed the best performance exceeding the IEEE

Table 2  
Performance evaluation of ML-based CSS

| A: ROC for performance evaluation, unsupervised-based CSS                                                                                                                                                                                                |                                                                                                                                              |                                                     |                                                              |                                      |                                                        |                                                                                                                                                                         |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|--------------------------------------------------------------|--------------------------------------|--------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Reference                                                                                                                                                                                                                                                | $P_D$ ( $P_{FA}$ )                                                                                                                           | Number of PUs (P)                                   | Number of SUs (N)                                            | SNR                                  | Number of training (M)/ testing samples                | Performance improvement                                                                                                                                                 |
| [10]                                                                                                                                                                                                                                                     | 0.98 (0.1)                                                                                                                                   | 1                                                   | 3                                                            | -12 dB                               | 500/500                                                | SNR+, N+, M+, k-, depends on channel type                                                                                                                               |
| [11]                                                                                                                                                                                                                                                     | 0.81 (0.1) GMM_E-val,<br>0.65 (0.1) GMM_E-vec<br>0.78 (0.1) k-mean_E-val,<br>0.73 (0.1) k-mean_E-vec                                         | 1                                                   |                                                              | -10 dB                               | 200 samples                                            | SNR+, N+, M+ ( $\geq 4$ ),<br>E-val does better                                                                                                                         |
| [12]                                                                                                                                                                                                                                                     | 0.97 (0.1) IMSE,<br>0.94 (0.1) IMME                                                                                                          | 1                                                   | 4                                                            | -15 dB                               | 1000/1000                                              | SRN+, N+, IMSE<br>does better                                                                                                                                           |
| [13]                                                                                                                                                                                                                                                     | 0.98 (0.1)                                                                                                                                   | 1                                                   | 10                                                           | -15 dB                               | 1000/1000                                              | SNR+, N+, M+                                                                                                                                                            |
| [17]                                                                                                                                                                                                                                                     | 0.89 (0.3)                                                                                                                                   | 4                                                   | 30                                                           | -15 dBW (PU TX power)                | 2000 samples                                           | SNR+, N+, M+, P-                                                                                                                                                        |
| [24]                                                                                                                                                                                                                                                     | 0.96 (0.1) DMEAE,<br>0.94 (0.1) DMM                                                                                                          | 1                                                   | 4                                                            | -18 dB                               | 500/500                                                | SNR+, N+, DMEAE<br>does better                                                                                                                                          |
| [19]                                                                                                                                                                                                                                                     | 0.96 (0.1) clts<br>= 9                                                                                                                       | 8<br>(2 active)                                     | 30                                                           | 0 dB<br>(-90 dBm noise floor)        | 100 simulation<br>scenarios                            | SNR+, cluster size (number of<br>SUs)+, number of clusters-                                                                                                             |
| [21]                                                                                                                                                                                                                                                     | 0.98 (0.1)                                                                                                                                   | 8<br>(2 active)                                     | 9                                                            | 50 mW (PU TX power)                  |                                                        | N+, P-, PU TX power-                                                                                                                                                    |
| [15]                                                                                                                                                                                                                                                     | 0.99 (0.025)                                                                                                                                 | 10 sub-bands<br>4 occupied                          |                                                              | -13 dB                               | 300 samples<br>(5000 simulation<br>trails)             | SNR+, a little affected by noise<br>uncertainly, affected considera-<br>bly by unequal noise variances,<br>affected considerably by the<br>number of occupied sub-bands |
| [25]                                                                                                                                                                                                                                                     | 0.90 (0.1) strategy 1<br>0.75 (0.01) strategy 2                                                                                              | 1                                                   |                                                              | -10 dB                               | 5000 (observation size)                                | SNR+, M+                                                                                                                                                                |
| B: ROC for performance evaluation, supervised-based CSS                                                                                                                                                                                                  |                                                                                                                                              |                                                     |                                                              |                                      |                                                        |                                                                                                                                                                         |
| Reference                                                                                                                                                                                                                                                | $P_D$ ( $P_{FA}$ )                                                                                                                           | Number of PUs (P)                                   | Number of SUs (N)                                            | SNR                                  | Number of training (M)/ testing samples                | Performance improvement                                                                                                                                                 |
| [5]                                                                                                                                                                                                                                                      | 0.96 lin,<br>0.94 poly                                                                                                                       | 1                                                   | 9                                                            | 300 mW (PU TX power)                 | 800/600                                                | SNR+, N+, M+, affected a little<br>by the kernel<br>function, $P_{FA}$ =0.05                                                                                            |
| [4]                                                                                                                                                                                                                                                      | 0.84 (0.1)                                                                                                                                   | 2                                                   | 25                                                           | 200 mW (PU TX power)                 | 560 samples                                            | SNR+                                                                                                                                                                    |
| [6]                                                                                                                                                                                                                                                      | 0.91 (0.1)                                                                                                                                   | 1                                                   | 32                                                           | -164 dBm/Hz (noise<br>power density) | 200 sample                                             | SNR+, N+, M+                                                                                                                                                            |
| [20]                                                                                                                                                                                                                                                     | 0.89 (0.1)                                                                                                                                   | 2                                                   | 5                                                            | -15 dB                               | 400/600                                                | SNR+, N+, M+                                                                                                                                                            |
| [14]                                                                                                                                                                                                                                                     | 0.92 (0.1) lin/<br>second-order-ED<br>0.90 (0.1) lin-CFD,<br>0.86 (0.1) second-order CFD<br>0.94 (0.1) lin-CD,<br>0.90 (0.1) second-order CD |                                                     | 5                                                            | -10 dB<br>-18 dB<br>-14 dB           | 200 bits<br>(observation<br>window size)               | SNR+, N+, window<br>size+                                                                                                                                               |
| [16]                                                                                                                                                                                                                                                     | 0.99 (0.1)                                                                                                                                   |                                                     |                                                              | -12 dB                               | 2000 samples<br>(1000 of them with<br>noise only)      | SNR+, almost same with<br>different OFDM signals                                                                                                                        |
| [26]                                                                                                                                                                                                                                                     | 0.90 (0.022) FM broadcast<br>0.82 (0.02) GSM-900 DL<br>0.96 (0.013) DCS-1800 DL<br>0.97 (0.019) UHF television                               |                                                     |                                                              | -14 dB                               | 500                                                    | SNR+, M+, affected by<br>different types of<br>radio technology                                                                                                         |
| [28]                                                                                                                                                                                                                                                     | 0.97 (0.02)                                                                                                                                  | 1                                                   | 8 number of<br>antennas                                      | -18 dB                               | 2000 samples                                           | SNR+, M+, P-                                                                                                                                                            |
| C: Throughput for performance evaluation                                                                                                                                                                                                                 |                                                                                                                                              |                                                     |                                                              |                                      |                                                        |                                                                                                                                                                         |
| Reference                                                                                                                                                                                                                                                | Throughput                                                                                                                                   | Training/testing samples<br>or number of iterations | Throughput<br>improvement                                    | The reference                        | Notes                                                  |                                                                                                                                                                         |
| [27]                                                                                                                                                                                                                                                     | 37% (SNR = -5 dB)<br>33% (SNR = -15 dB)                                                                                                      | 500/2000                                            | Fixed when the battery<br>capacity $\geq 1500$ packets, SNR+ | Max/min of PU TX<br>power            | 2/2 (number of PU/number of SU)                        |                                                                                                                                                                         |
| [18]                                                                                                                                                                                                                                                     | 83% ( $\epsilon = 0.1$ )<br>60% ( $\epsilon = 0.5$ )                                                                                         | 2000                                                | Convergence at<br>200 iterations                             | The ideal policy                     |                                                        |                                                                                                                                                                         |
| [7]                                                                                                                                                                                                                                                      | $\sim 1.7$ normalized<br>accumulative rewards                                                                                                | 5000                                                | Convergence at<br>5000                                       | Other schemes                        | The discount factor $\gamma = 0.4$<br>$\epsilon = 0.8$ |                                                                                                                                                                         |
| <p>Legend:</p> <p>PU TX power is an abbreviation of the PU's transmitted power</p> <p>E-val and E-vec are the eigenvalue and the eigenvector, respectively</p> <p>+ sign means increased by an increase</p> <p>- sign means increased by an decrease</p> |                                                                                                                                              |                                                     |                                                              |                                      |                                                        |                                                                                                                                                                         |

802.22 standard ( $\geq 90\%$ ) at the lowest SNR ( $-18$  dB). K-medoids with an improved ratio of the maximum and minimum eigenvalues and the difference between the maximum eigenvalue and the average eigenvalue [12], fuzzy c-means with geodesic distance [13], and kernel fuzzy c-means with max energy [17] showed the second lowest SNR result ( $-15$  dB). However, kernel fuzzy c-means with max energy failed to meet the requirement of the IEEE 802.22 standard related to  $P_D$  and  $P_{FA}$  (0.89 and 0.3 respectively). This is caused by the fact that the system model of the scheme in question is more complicated, as there are four PUs that are active in the CRN.

Strategy 1 of the blind continuous hidden Markov model with the continuous wavelet transform [25] achieved the requirement of the IEEE 802.22 standard related to  $P_D$  percentage, but failed to achieve it with regard to low SNR. However, its performance may be improved by increasing the PU's transmit power or by increasing the number of collaborating SUs. GMM, k-means with the eigenvalue and eigenvector [11] and the strategy 2 of the blind continuous hidden Markov model [25] failed to achieve the requirements of the IEEE 802.22 standard concerning  $P_D$  percentage ( $< 0.9$ ) or SNR ( $-10$  dB).

Supervised-based CSS schemes also showed good performance related to ROC curve, as shown in Table 2b. Supervised SVM with beamformer-aided scheme and linear classifier with the CFD method exhibited the best performance, exceeding the IEEE 802.22 standard ( $\geq 90\%$ ) at the lowest SNR ( $-18$  dB). On the contrary, 2-order polynomial classifier with the CFD method, as well as linear and 2-order polynomial classifier with the ED method failed to achieve the requirements of the IEEE 802.22 standard (0.86 in the case of  $P_D$  percentage and  $-10$  dB in the case of SNR, respectively). However, performance of 2-order polynomial classifier with the CFD method may be improved by increasing the PU's transmitting power. 2-phase SVM [4] and multi-class SVM [20] with the ED method failed to achieve the requirements of the IEEE 802.22 standard concerning  $P_D$  percentage ( $< 0.90$ ), despite multi-class SVM with the ED method achieving the second lowest SNR. Linear and 2-order polynomial classifier with the CD method [14], as well as the back propagation neural network (with ED and likelihood ratio test statistic as input samples) that is employed in different radio technologies, achieved the third lowest SNR ( $-14$  dB), showing superior performance in terms of  $P_D$  percentage as well ( $\geq 0.90$  despite the use of GSM-1800 DL radio technology).

In general, the superior performance of several unsupervised- and supervised-based CSS approaches is usually related to the small scale of CRNs. Small scale networks are less complex and less prone to signal interference. So, they are expected to offer higher performance compared to large-scale networks. In principle, overall performance of unsupervised- and supervised-based CSS is mainly affected by four different factors. First, the PU's transmission power (the higher the better). Second, the number of collabo-

rating SUs (the higher the better). Third, the number of PUs present (the lower the better), and finally, the number of training samples. A higher number of samples will lead to better training results in general.

## 6.2. Throughput for Performance Evaluation

As stated above, performance was also measured using throughput [27], [18], [7] – as shown in Table 2c. The supervised multi-class SVM with ED methods [27] achieved 37% and 33% of the maximum throughput at  $-5$  dB SNR and  $-15$  dB SNR, respectively. The maximum throughput may be achieved by ensuring that the PU is sending with the highest possible transmission power. The sensing approach based on the  $\epsilon$ -greedy policy [18] achieved the result of 83%, compared to the result of an ideal policy equaling  $\epsilon = 0.1$ . Ideal policy assumes that the sub-bands with the highest instantaneous throughput may be selected, and the highest instantaneous throughputs may then be found. Finally, replicated Q-learning-based sub-band selection [7] achieved a normalized accumulated reward of approximately 1.7, representing a throughput that is comparable with that of several sub-band selection policies, such as Bellman-optimality equation and Markov decision process. However, in reinforcement-based CSS schemes, throughput converges as the system trains more, meaning that these schemes cannot be applied in real time.

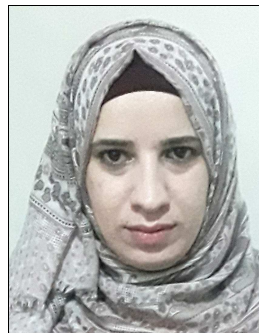
In general, unsupervised-based CSS schemes offer reasonable performance, but their accuracy is slightly lower compared to other approaches [5]. Supervised-based CSS schemes are more accurate than others, but generate some overhead during the training phase, because of the labeling process [6]. Reinforcement-based CSS schemes lead to several challenges, such as high computational complexity and the policy needed to operate in real-time, so this type is rarely used when proposing new ML-based CSS schemes [7].

## 7. Conclusion

This paper considered the main types of machine learning algorithms and categorized the proposed ML-based CSS schemes into three classes, namely unsupervised-based CSS, supervised-based CSS, and reinforcement-based CSS. It also categorized the mathematical algorithms used for feature extraction, dividing them into three types: energy detection-based feature extraction, cyclostationary-based feature extraction, and signal processing-based feature extraction. Finally, analysis of the results of several proposed schemes showed that unsupervised-based CSS is less complex but a bit less accurate than other approaches. On the other hand, supervised-based CSS offered excellent accuracy, but caused a slight overhead during the training phase. Reinforcement learning offers reasonable accuracy and improves throughput, but may be hard to apply in real time and requires high computational complexity.

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# Performance Improvement for Vehicular Communications Using Alamouti Scheme with High Mobility

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<https://doi.org/10.26636/jtit.2020.140120>

**Abstract**—The IEEE 802.11p standard is the basic protocol for wireless access in a vehicular environment (WAVE), providing high throughput for multimedia and high quality for vehicular transmissions. However, IEEE 802.11p fails to offer any multi-antenna approaches. In this paper, a multiple-input single-output (MISO) implementation with orthogonal frequency division multiplexing (OFDM), aiming to improve the performance of IEEE 802.11p, is proposed. The authors investigate the impact of time-varying channel on the performance of Alamouti space-time block codes (STBC) in OFDM systems. The Alamouti STBC approach shows good performance in slow time-varying environments, while its Alamouti space frequency block codes (SFBC) counterpart performs better over fast time-varying environments. An adaptive switching scheme is proposed to select appropriate space-block coding (STBC or SFBC) in vehicular channels with high mobility levels. It is shown that the proposed adaptive scheme provides better performance compared with traditional space-block codes.

**Keywords**—IEEE 802.11p, MIMO, MISO, OFDM, SFBC, STBC, vehicular channel.

## 1. Introduction

In recent years, vehicular technologies have been widely used to improve safety. Today, cameras and radars reduce accident rates and improve road safety, but safe and autonomous driving systems require high quality for communications between vehicles and their environment. The IEEE 802.11p standard is designed for vehicular networks and is based on OFDM modulation in the 5.9 GHz band. The traditional IEEE 802.11a standard was developed primarily for Wi-Fi wireless networks, characterized by low mobility of the receiver/transmitter [1]. The IEEE 802.11p variety is developed for use in typical outdoor vehicular networks with high mobility rates [2].

The emergence of vehicular technologies observed these days creates new requirements for wireless communications, such as high data rates for multimedia applications and high quality links for real time systems. Therefore, multiple-input multiple-output (MIMO) transmissions over wireless multipath channels will be a promising so-

lution [3]. While the MIMO technique is already used in indoor wireless LAN (Wi-Fi) and cellular networks, such as LTE and WiMAX, the IEEE 802.11p comes without any particular MIMO scheme. However, MIMO approaches may considerably improve the robustness of IEEE 802.11p. MIMO schemes may be classified as space diversity techniques used to improve link quality, and spatial multiplexing techniques used to improve data rates [1].

Space diversity MIMO techniques are widely used to improve link quality while dealing with channel fading effects. The basic idea behind space diversity is that multiple signal copies (redundant symbols) are transmitted. This significantly reduces the detection error rate at the receiver side by improving efficiency of the detection process. Such an approach improves the quality of wireless communications by transmitting more than one copy of the signal over multiple independent fading channels, while keeping the total transmitted power constant and while maintaining the same bandwidth (as in a single antenna scheme). The probability that all signal copies fall in deep channel fading may be greatly reduced compared to the single antenna scheme [4]. The typical examples of MIMO space diversity are time diversity, where the signal is repeated over successive time slots, and frequency diversity [5].

Space time block code (STBC) schemes are a promising MIMO space diversity approach, where diversity is provided over the space-time dimensions. It has been proved that MIMO STBC schemes improve wireless link quality without increasing transmitted power or frequency bandwidth [6]. However, MIMO STBC techniques assume that the transmission is made over a time-invariant channel, for the entire duration of the STBC block. This assumption is not always valid for wireless communications over fast time-variant channels, such as vehicular channels with high mobility of solo transmitters/receivers. Therefore, the performance of STBC will be seriously degraded in wireless time-variant channels, where the channel does not remain constant over the successive time slots.

The first STBC scheme, known as the  $2 \times 1$  Alamouti scheme, involves two transmit antennas and one receive antenna. It has been shown that  $2 \times 1$  Alamouti outper-



forms the traditional  $1 \times 1$  single antenna scheme, maintaining the same transmit power and bandwidth. Alamouti codes provide full diversity and orthogonality [6]. Various STBC codes have been designed so far for massive MIMO technologies, but the Alamouti  $2 \times 1$  scheme has been widely used because it is the only STBC approach capable of achieving full diversity and orthogonality.

The combination of MIMO and OFDM has attracted a lot of attention over the recent years. It is considered to be a promising technique for wireless high-speed data transmissions over mobile multipath channels [7]. Many wireless standards, such as WiMAX and Wi-Fi, are based on MIMO OFDM. The combination of MIMO and OFDM creates space-frequency block codes (SFBC) in MIMO OFDM systems. The difference between SFBC and STBC techniques consists in the fact that in SFBC, the code is done over frequency, while in STBC, the code is done over time [8].

This work investigates the performance of STBC and SFBC codes in vehicular communications in time and frequency selectivity domains. STBC's assumption of a time-invariant channel (over successive time slots) is not necessarily valid for high mobility vehicular channels. Similarly, SFBC's assumption of a frequency-invariant channel (for neighboring subcarriers) is not necessarily valid over mobile multipath channels (frequency-selective channels).

Adaptive switching has been proposed in order to overcome the problem of time and frequency selectivity in vehicular communications. The idea is based on dynamic switching between STBC and SFBC, according to channel conditions. It has been proved that the proposed scheme performs better than STBC and SFBC schemes applied on their own. Such adaptive switching has been already proposed earlier in [9]. However, the switching criteria adopted in [9] require perfect channel state information and a known correlation matrix at the transmitter side. This requirement cannot be satisfied in time-variant channels, such as the vehicular channel. Otherwise, implementation of the proposed adaptive switching might require periodic feedback from the receiver. Hence, the switching criteria proposed in [9] cannot be implemented for vehicular channels with high mobility. The switching criterion proposed here does not require any feedback and, hence, may be easily implemented at the transmitter side.

The presented development of MISO OFDM is aimed to enhance the IEEE 802.11p standard by improving adaptive switching between Alamouti STBC and SFBC schemes. The switching criterion is based on normalized maximum Doppler frequency and normalized delay spread. Being different from [9], the system requires only an estimation of channel delay spread and maximum Doppler frequency. Thanks to this, the switching decision may be easily made at the transmitter side.

This paper is organized as follows. In Section 2, the basic structure of IEEE 802.11p and the design of STBC/SFBC schemes in MISO OFDM systems are described. In Section 3, time frequency selectivity in vehicular channels is

analyzed. The proposed adaptive switching design is developed in Section 4. The simulation results are shown in Section 5.

## 2. System Description

IEEE 802.11p is designed for vehicular networks and is an approved amendment to the IEEE 802.11a standard [10]. The physical layer of IEEE 802.11p is similar to the one used in IEEE 802.11a [11], [12], but the updated version has a lower overhead in order to provide low delays, real time data exchange and fast exchange of safety messages. In addition, IEEE 802.11p uses a narrower bandwidth (10 MHz channels).

### 2.1. Architecture of OFDM IEEE 802.11p

The physical layer of IEEE 802.11p is based on OFDM modulation employing 64 subcarriers. In wireless communications, channel state information is required by the receiver. Meanwhile, in OFDM transmissions, pilot subcarriers are used for wireless channel estimation. Therefore, 48 of the 64 subcarriers are used for data transmission and 4 subcarriers for pilot transmission (channel estimation purposes). IEEE 802.11p offers a data transmission rate of 3, 4.5, 6, 9, 12, 18, 24 and 27 Mbps (Table 1) [13], [14]. The frequency range used is 5.850-5.925 GHz, which is divided into 7 of 10 MHz channels. The standard is based on four complex modulations: BPSK, QPSK, 16-QAM and 64-QAM. The modulation scheme used depends on the required data rate and on wireless channel conditions [15].

Table 1  
IEEE 802.11p parameters

| Parameters              | Values                            |
|-------------------------|-----------------------------------|
| Data subcarriers        | 48                                |
| Pilot subcarriers       | 4                                 |
| Total subcarriers       | 52                                |
| IFFT/FFT size           | 64                                |
| TTFT: IFFT/FFT period   | $6.4 \mu s$                       |
| TGI: GI duration        | $1.6 \mu s$ (TTFT/4)              |
| T: symbol period        | $8.0 \mu s$ (TTFT + TGI)          |
| Channel spacing         | 10 MHz                            |
| Signal bandwidth        | 8.3 MHz                           |
| Modulation              | BPSK, QPSK, 16QAM, 64QAM          |
| Data rate               | 3, 4.5, 6, 9, 12, 18, 24, 27 Mbps |
| Subcarrier spacing      | 156.2 KHz                         |
| Number of guard samples | 16                                |

### 2.2. MISO Alamouti STBC

The Alamouti STBC is considered to be an improved transmission scheme aiming to enhance the performance of wireless communications. MISO systems with STBC re-

quire less power than is needed by a single antenna system. The STBC technique expands the transmission into two dimensions: space (via many antennas) and time (by successive time slots) [6]. The Alamouti STBC scheme is based on the assumption of time-invariance of the channel during the STBC block (two successive time slots). Therefore, the assumption is not necessarily valid for wireless channels in high mobility applications. The Alamouti  $2 \times 1$  scheme is designed to achieve spatial diversity with two transmit antennas and one receive antenna (Fig. 1) [6].

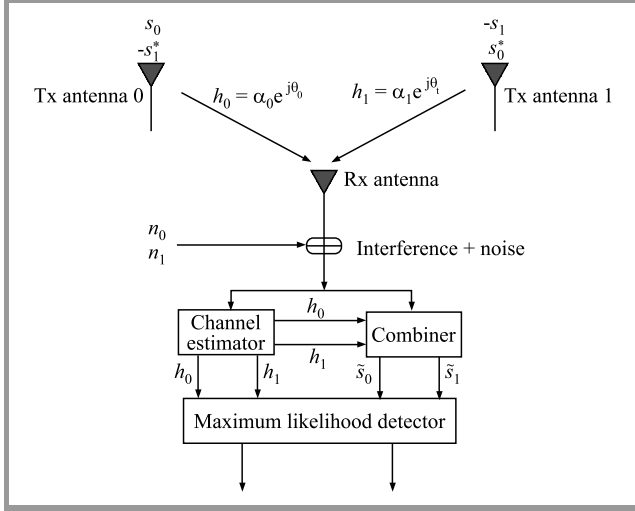


Fig. 1. Alamouti  $2 \times 1$  STBC.

The first antenna transmits symbols  $s_0$  at time  $t_0$  and  $-s_1^*$  at time  $t_0 + T$  respectively. Symbols  $s_1$  and  $s_0^*$  are simultaneously transmitted by the other antenna. Considering the Alamouti STBC assumption of a time-invariant channel between the two successive time slots, the symbols received at time  $t_0$  and  $t_0 + T$  may be expressed as:

$$r_0 = r(t_0) = s_0.H_0 + s_1.H_1 + w_0, \quad (1)$$

$$r_1 = r(t_0 + T) = -s_1^*.H_0 + s_0^*.H_1 + w_1. \quad (2)$$

The symbols received at time  $t_0$  and  $t_0 + T$  are given by Eqs. (1) and (2), where  $H_0$  and  $H_1$  are the channel coefficients,  $w_0$  and  $w_1$  are the receiver noise.

The great advantage of the Alamouti  $2 \times 1$  STBC scheme is that the received symbols may be recovered by a simple linear operation (without noise amplification) [6] (Fig. 1).

$$\tilde{s}_0 = H_0^*.r_0 + H_1.r_1^*, \quad (3)$$

$$\tilde{s}_1 = H_1^*.r_0 + H_0.r_1^*. \quad (4)$$

### 2.3. Alamouti STBC OFDM System

In this section, we consider a transmission sequence  $\{s_1, s_2, \dots, s_N\}$ . In the single antenna scheme, we will transmit symbol  $s_1$  at time  $t_0$  (first time slot), symbol  $s_2$  at time  $t_0 + T$  (second time slot),  $s_3$  at time  $t_0 + 2T$  (third time slot), etc. In the MISO  $2 \times 1$  Alamouti scheme, two symbols will be transmitted at the same time by two trans-

mit antennas. Alamouti suggested that in the first time slot we transmit symbols  $s_1$  and  $s_2$  from the first and second antenna, respectively, while in the second time slot – we transmit symbols  $-s_2^*$  and  $s_1^*$  [6].

Considering the OFDM system with  $N$  subcarriers, if we apply  $2 \times 1$  Alamouti STBC to our OFDM modulation, the encoded data will be transmitted over three dimensions: time, frequency (on subcarriers) and space (via many antennas) (Table 2).

Table 2  
 $2 \times 1$  STBC OFDM system basics

|           | Subcarrier | Time $t_0$ | Time $t_0 + T$ |
|-----------|------------|------------|----------------|
| Antenna 1 | 1          | $s_1$      | $-s_2^*$       |
|           | 2          | $s_3$      | $-s_4^*$       |
|           | ...        | ...        | ...            |
|           | $N$        | $s_{2N-1}$ | $-s_{2N}^*$    |
| Antenna 2 | 1          | $s_2$      | $s_1^*$        |
|           | 2          | $s_4$      | $s_3^*$        |
|           | ...        | ...        | ...            |
|           | $N$        | $s_{2N}$   | $s_{2N-1}^*$   |

The transmitted symbols at time  $t_0$  and time  $t_0 + T$  are:

$$\text{Antenna 1: } S_0 = [s_1, s_3, \dots, s_{2N-1}, -s_2^* - s_4^* - s_{2N}^*],$$

$$\text{Antenna 2: } S_1 = [s_2, s_4, \dots, s_{2N}, s_1^*, s_3^*, \dots, s_{2N-1}^*],$$

where  $S_0$  and  $S_1$  are the transmitted sequences from the first and second antenna, respectively,  $N$  is the number of subcarriers in OFDM modulation and  $2N$  represents the number of transmitted symbols at time  $t_0$  and time  $t_0 + T$ . The MISO OFDM scheme based on  $2 \times 1$  Alamouti STBC is shown in Fig. 2 [16].

Under the assumption of the time-invariant channel,  $H(n+1, k) = H(n, k)$ , the received signal may be represented in a matrix notation as [6], [7]:

$$R = H_0.S_0 + H_1.S_1 + W, \quad (5)$$

where

$$H_0 = [H_0(n, 1) \dots H_0(n, N), H_0(n+1, 1) \dots H_0(n+1, N)]^T,$$

$$H_1 = [H_1(n, 1) \dots H_1(n, N), H_1(n+1, 1) \dots H_1(n+1, N)]^T,$$

$$R = [R(n, 1) \dots R(n, N), R(n+1, 1) \dots R(n+1, N)]^T,$$

$R(n, k)$  is the received symbol loaded onto the  $k$ -th subcarrier from the  $n$ -th OFDM block,  $S_0$  and  $S_1$  are the transmitted symbols from the first and second antenna, respectively, and  $W$  is the noise vector. According to Eqs. (3) and (4), the recovered signals are:

$$\tilde{s}_i(n, k) = H_0^*(n, k)R(n, k) + H_1(n, k)R^*(n+1, k), \quad (6)$$

$$\tilde{s}_{i+1}(n, k) = H_1^*(n, k)R(n, k) - H_0(n, k)R^*(n+1, k), \quad (7)$$

where  $i$  refers to symbol number ( $i = 1, 3, 5, \dots, N-1$ ), knowing that  $2N$  symbols are transmitted during the two

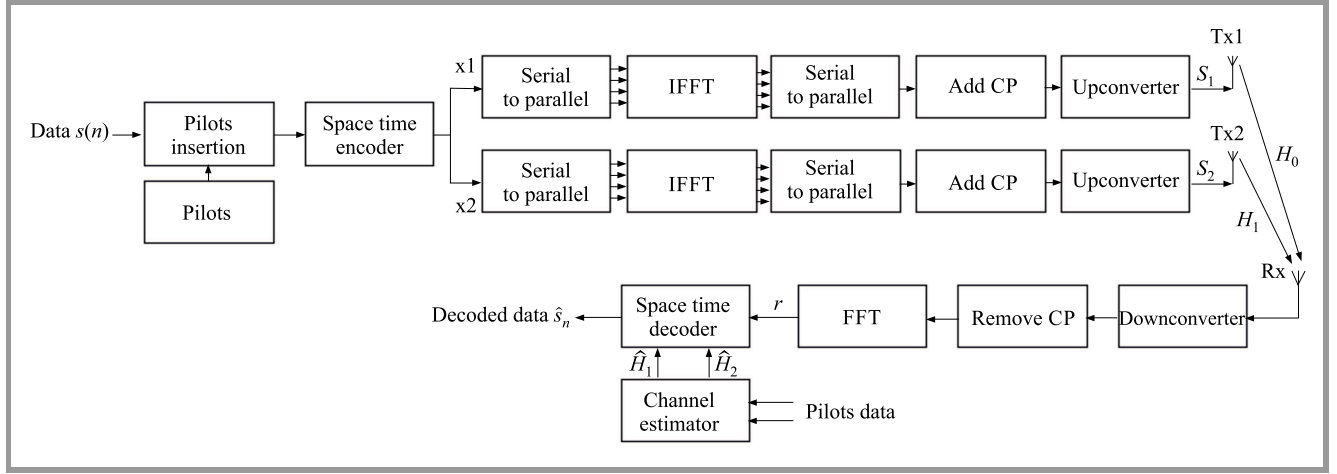


Fig. 2. Block diagram of a 2×1 antennas MISO OFDM system in STBC.

time slots (the first time slot starts at time  $t_0$  and the second time slot starts at time  $t_0 + T$ ),  $n$  refers to the  $n$ -th OFDM block and  $k$  refers to the  $k$ -th subcarrier ( $k = 1, 2, \dots, N$ ). Parameters  $H_0$  and  $H_1$  are the channel coefficients for antenna 1 and antenna 2, respectively. Knowing that channel estimation is required on the receiver side to estimate  $H_0$  and  $H_1$ , the receiver may produce  $\hat{s}_i$  and  $\hat{s}_{i+1}$ .

#### 2.4. Alamouti SFBC-OFDM Scheme

In OFDM systems, channel frequency response remains almost invariant on neighboring subcarriers of the same OFDM symbol. Alamouti space-frequency block coding (SFBC) transmits symbols on neighboring subcarriers over the frequency domain rather than the time domain that is used in Alamouti STBC. OFDM transforms a frequency-selective channel into several flat fading channels. Then, expansion of the transmission to the space-frequency dimension becomes an interesting opportunity with SFBC schemes [16], [17].

Alamouti STBC suffers from susceptibility to fast fading variation over time. Therefore, the SFBC design is an attractive approach for robust transmissions over time selective channels. In SFBC OFDM, transmission redundancy is achieved over both space and frequency, as for each OFDM symbol, neighboring subcarriers  $k$  and  $k + 1$  ( $k = 1, \dots, N$ ) are used for data encoding over the space-frequency dimension. Instead of transmitting one ( $n$ -th) OFDM symbol  $S(n) = [s_1(n), s_2(n), \dots, s_N(n)]$ , in two OFDM symbols by two transmit antennas are used:

$$S_1(n) = [s_1(n), -s_2^*(n), \dots, s_{N-1}(n), -s_N^*(n)] ,$$

$$S_2(n) = [s_2(n), s_1^*(n), \dots, s_N(n), s_{N-1}^*(n)] ,$$

$S_1(n)$  is transmitted from the first antenna and  $S_2(n)$  is transmitted simultaneously from the other antenna (Fig. 3).

$$R_k = H_0(f_k) \cdot s_k + H_1(f_k) \cdot s_{k+1} + W_k , \quad (8)$$

$$R_{k+1} = H_0(f_{k+1}) \cdot s_{k+1}^* + H_1(f_{k+1}) \cdot s_k^* + W_{k+1} . \quad (9)$$

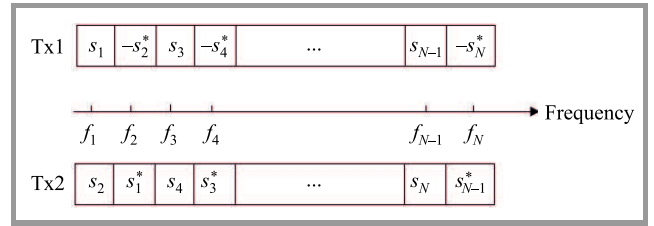


Fig. 3. 2×1 Alamouti SFBC design.

Under the assumption of invariant channel over the neighboring subcarriers [16], we can assume that  $H_0(f_k) = H_0(f_{k+1})$  and  $H_1(f_k) = H_1(f_{k+1})$ , and Eqs. (8) and (9) may be simplified as:

$$\tilde{s}_k = H_{0k}^* \cdot R_k + H_{1k} R_{k+1}^* , \quad (10)$$

$$\tilde{s}_{k+1} = H_{1k}^* R_k - H_{0k} R_{k+1}^* , \quad (11)$$

Equations (10) and (11) are similar to (1) and (2), hence maximum diversity for this SFBC design is achieved.

### 3. Vehicular Channel

The multipath channel is an emerging field of research due to its great impact on wireless communications. Wireless channels are time-frequency selective. The time or frequency selectivity refers to the variation of the channel as a function of time or frequency (Fig. 4). Formally, time-frequency selectivity is characterized by maximum delay spread  $\tau_{\max}$ , that depends on wave reflections from obstacles, and maximum Doppler frequency  $f_{d\max}$ , that depends on transmitter/receiver mobility. Time selectivity of a channel may be measured by the maximum Doppler frequency, and its frequency selectivity may be evaluated by the maximum delay spread [17]. In other words, a wireless channel with a long delay spread is frequency selective. These types of channels exhibit lower bandwidth coherence. Similarly, a wireless channel with a high Doppler frequency is time selective and exhibits lower coherence time.

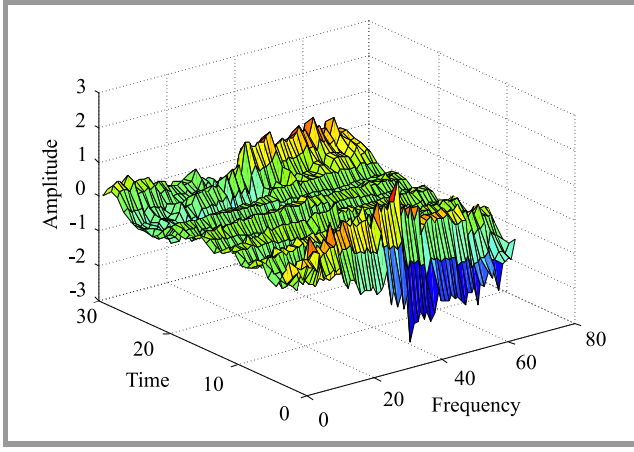


Fig. 4. Time frequency selective channel. (For color pictures see the electronic version of the paper.)

### 3.1. Vehicular Channel Model

In the time domain, the wireless multipath channel is described by its channel impulse response [18], [19]:

$$h(t, \tau) = \sum_{l=1}^L \alpha_l(t) \delta[\tau - \tau_l(t)] , \quad (12)$$

where  $L$  refers to the number of pathways of the multipath channel,  $\alpha_l$  is the  $l$ -th path complex gain and  $\tau_l$  is the  $l$ -th path propagation.

In discrete-time systems, the signal is transmitted in regular time slots  $T_s$ , where  $T_s$  refers to the sampling period. Under the assumption of fixed and known number of channel paths  $L$ , the channel impulse response may be written as:

$$h(t, \tau) = \sum_{l=1}^L \alpha_l(t) \delta(\tau - \tau_l T_s) . \quad (13)$$

$\alpha_l(t)$  the  $l$ -th path channel gain are function of time, this variation depends on Doppler frequency  $f_d = v_m \cdot f \cdot \cos(\theta)/c$ , where  $v_m$  is the transmitter/receiver velocity,  $\theta$  is the arrival azimuth of the electromagnetic wave,  $f$  is the signal frequency and  $c$  the electromagnetic wave speed.

### 3.2. Time Selectivity in Vehicular Channels

In vehicular channels, time-selective fading, known as channel time selectivity, is a consequence of the Doppler effect experiences due to the mobility of the transmitter/receiver. Time selectivity of a channel is usually evaluated by its time coherence  $T_c$  which is used to characterize the time-varying nature of channel attenuation [17]:

$$T_c \approx \frac{9}{16\pi f_{d,\max}} , \quad (14)$$

where the maximum Doppler frequency  $f_{d,\max}$  is:

$$f_{d,\max} = \frac{v_m f}{c} . \quad (15)$$

In OFDM systems, a wireless channel is considered time-selective or fast time-varying when the channel's coherence time is lower than OFDM block duration  $T_c < T$  [19]. Figure 5 shows results of simulations for different vehicle speeds over a Rayleigh fading channel. At varying speeds ranging between 30, 60 and 220 km/h, one can observe a significant impact on the channel's time selectivity.

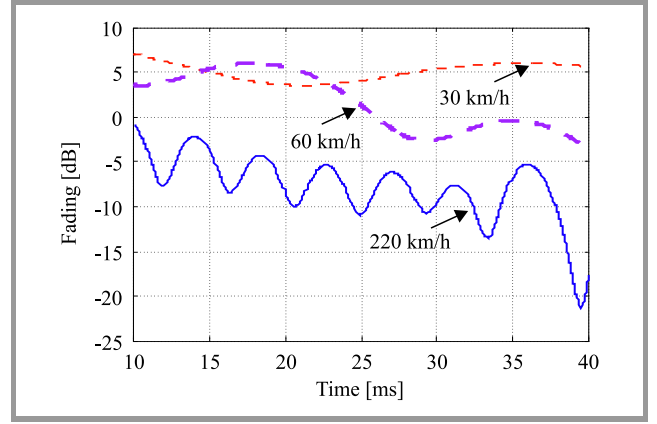


Fig. 5. Wireless fading channel for varying vehicle speeds.

At the operating frequency of 5.9 GHz, the time duration of an OFDM symbol is 8  $\mu$ s (IEEE 802.11p).

If the expected speed of the vehicle is 30 km/h, the Doppler shift is given by 163.88 Hz and, thus, the coherence time is in the order of 1.1 ms. For 60 km/h, the Doppler shift is 327.77 Hz, and the coherence time is 0.5 ms. At 220 km/h, the Doppler shift is increases to 1201.9 Hz and, thus, the coherence time is in the order of 0.1 ms.

Time selectivity is caused by the mobility of the vehicle transmitter/receiver due to the induced Doppler shift that might easily reach 1200 Hz at the carrier frequency of 5.9 GHz [20]. This large value may harm the performance of the system, especially if MISO Alamouti STBC systems are used. The Alamouti assumption of an invariant channel over the STBC block is not valid here. The channel's time selectivity is a serious challenge in MISO STBC schemes used for high mobility vehicular communications.

### 3.3. Frequency Selectivity in Vehicular Channels

In vehicular communication, the wireless channel environment is characterized by the presence of electromagnetic wave scattering. Due to wave reflections, refractions and diffractions, the received signal is a superposition of several delayed copies of the transmitted signal, and multipath propagation leads to frequency selectivity of the wireless channel.

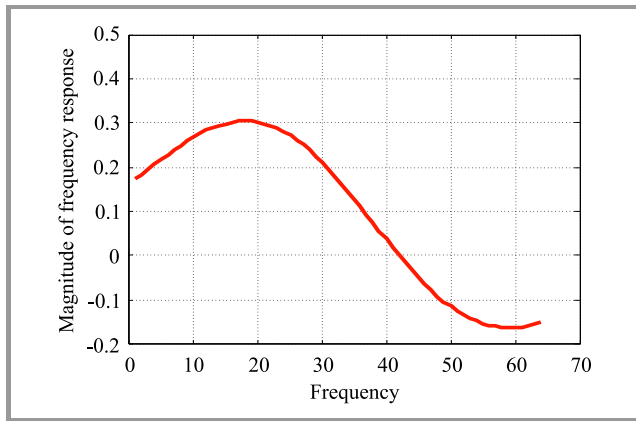
The channel is considered as frequency flat fading when the channel delay spread is much lower than the signal sampling period. Analogically, the wireless channel is frequency flat fading when it has only one pathway ( $\tau_{\max} = 0$ ).

Channel frequency selectivity is evaluated by the coherence bandwidth  $B_c$ , measured for the frequency range over which the channel is frequency flat fading.

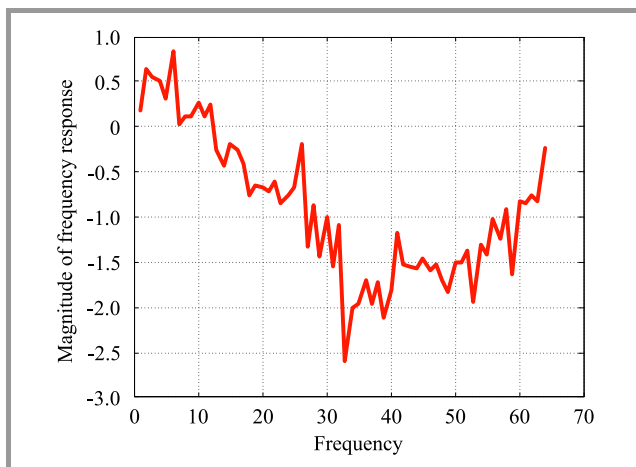
In OFDM systems, the channel is considered severely frequency selective when the coherence bandwidth is lower than the bandwidth of two subcarriers  $\Delta f < B_c < 2\Delta f$ , where  $B_c$  is the coherence bandwidth and  $\Delta f$  is the subcarrier bandwidth in the OFDM system. An OFDM wireless channel is considered non-frequency-selective when  $B_c > 2\Delta f$ , and when the neighboring subcarriers have the same frequency response. In the Rayleigh fading model, the coherence bandwidth is approximated as [17]:

$$B_c \approx \frac{1}{\tau_{\max}}. \quad (16)$$

A wireless channel is considered non-frequency-selective (or moderately selective) when the signal bandwidth is lower than the channel coherence bandwidth  $B < B_c$ , knowing that  $B = 1/T_s$  and  $T = N$ , where  $T_s$  and  $T$  refer to the sampling period and OFDM block duration, respectively. In other words, the channel is non-frequency-selective, when its maximum delay spread  $\tau_{\max}$  is lower than the sampling period  $T_s$  ( $T_s > \tau_{\max}$ ).



**Fig. 6.** Frequency response of a vehicular channel when delay spread is  $3T_s$  ( $T_s = 0.1 \mu s$ ).



**Fig. 7.** Frequency response of a vehicular channel when delay spread is  $32T_s$  ( $T_s = 0.1 \mu s$ ).

A wireless channel is considered frequency-selective when the channel's coherence bandwidth  $B_c$  is lower than signal bandwidth  $B$ , and then  $T_s > \tau_{\max}$ .

Frequency selectivity is an important parameter of vehicular channels due to the presence of many scatterers in the propagation environment [20]. Frequency response of the channel is calculated by converting its impulse response to the frequency domain. As shown in Figs. 6 and 7, vehicular channels offer high frequency selectivity when delay spread is higher. In this simulation, coherence bandwidth equals 3.3 MHz in Fig. 6, and 0.5 MHz in Fig. 7. It is important to note that coherence bandwidth decreases when the vehicular environment is saturated with scatterers (very high delay spread), i.e. as the vehicle is moving along a highway. Therefore, coherence bandwidth may be lower than 0.1 MHz.

In IEEE 802.11p, subcarrier spacing is in the order of 0.1 MHz ( $\Delta f = 10 \text{ MHz}/64$ ). A vehicular channel is considered severely frequency-selective when delay spread is greater than  $3.2 \mu s$  ( $\tau_{\max} > 32T_s \rightarrow B_c < 2\Delta f$ ). In severely frequency-selective channels, the assumption of Alamouti SFBC invariant channel transfer function over adjacent subchannels is not valid, unfortunately, and the frequency selective channel degrades the performance of SFBC. Therefore, in order to design robust MISO schemes, the choice of SFBC or STBC coding for vehicular channels will be based on channel conditions.

## 4. Proposed Adaptive Switching Technique

Since STBC and SFBC schemes show contradicting behaviors over time frequency-selective channels, a switching technique is proposed in order to select the appropriate transmission scheme (STBC or SFBC) according to vehicular channel characteristics.

### 4.1. Time Frequency Correlation Strength

Channel fading is a consequence of multipath wave propagation and the received signal is a superposition of different waves originating from different paths. A vehicular channel may be modeled statistically as a Ricean or Rayleigh fading channel. These two models describe the received power correlation over a multipath channel. If there is one dominant wave with line-of-sight (LOS) propagation between the sender and the receiver, and several indirect waves, Ricean fading channel occurs. If no LOS propagation is possible and multiple indirect waves are available only, Rayleigh fading channel occurs [21]. The Ricean fading model is similar to the Rayleigh fading model, except that in the Ricean model LOS propagation is present and a dominant wave component is transmitted directly from the sender to the receiver.

Here, a vehicular Rayleigh fading channel model is considered and channel selectivity is evaluated by analyzing channel correlation. The temporal selectivity of the channel is examined by computing time correlation strength  $\rho_t$ .



Furthermore, frequency selectivity is investigated by computing frequency correlation strength  $\rho_f$ . The basic idea of adaptive switching is to select an appropriate transmission scheme (STBC or SFBC) for a given vehicular channel. This switching process is governed by the channel's time-frequency selectivity, and then the time and frequency correlation strength of the vehicular channel are needed in order to choose the appropriate transmission mode.

In the OFDM system, the time correlation strength  $\rho_t$  is evaluated by measuring correlation for the same subcarrier between adjacent OFDM blocks, while frequency correlation strength  $\rho_f$  is evaluated by measuring correlation between adjacent subcarriers for the same block. The strength of time and frequency correlations is compared in order to select the appropriate transmission scheme. Time and frequency correlation strength is evaluated for OFDM transmissions over the Rayleigh fading channel in the following manner [22]:

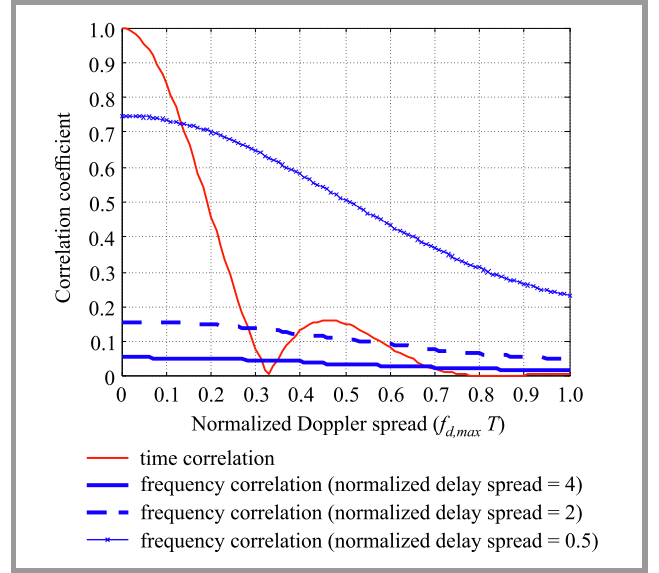
$$\rho_t = \frac{1}{N^2} \sum_{l=-N+1}^{N-1} (N-|l|) J_0 \left[ 2\pi f_d T \left( 1 + \frac{G}{N} + \frac{l}{N} \right) \right], \quad (17)$$

$$\rho_f = \frac{(1-\lambda) \left( 1 - \lambda^M e^{-\frac{j2\pi M}{N}} \right)}{(1-\lambda^M) \left( 1 - \lambda e^{-\frac{j2\pi M}{N}} \right)} \frac{1}{N^2} \sum_{l=-N+1}^{N-1} (N-|l|) J_0 \left[ \frac{2\pi f_d T l}{N} \right], \quad (18)$$

where  $\lambda = e^{-\frac{T}{\tau_{\max}}}$ ,  $N$  is number of OFDM subcarriers,  $G$  is number of OFDM guard samples and  $T$  is the OFDM block duration.  $J_0[\cdot]$  is the Bessel function of order zero.

#### 4.2. Dynamic STBC/SFBC Allocation in Vehicular Communications

Here, a comparative and correlational study of time and frequency correlations is presented. Figure 8 shows the magnitude of time correlation strength  $|\rho_t|$  and frequency correlation strength  $|\rho_f|$  in terms of normalized Doppler spread  $f_{d,\max} \cdot T$ , and normalized delay spread  $\frac{\tau_{\max}}{T}$ , where  $f_{d,\max}$  is the maximum Doppler frequency,  $\tau_{\max}$  is the channel's maximum delay spread and  $T$  is the OFDM symbol duration. The time correlation strength is governed by one parameter, i.e. Doppler spread  $f_{d,\max} \cdot T$ , see Eq. (17). From Fig. 8, we can see that time correlation is strong ( $|\rho_t| \gg |\rho_f|$ ) when normalized Doppler spread is considerably lower than one ( $f_{d,\max} \cdot T \ll 1$ ), which is typical of a slow time-varying channel, such as the vehicular channel with  $f_{d,\max} \cdot T = 0.01$  or  $f_{d,\max} \cdot T = 0.05$ . In these channel conditions, the STBC scheme will perform better than its SFBC counterpart. However, time correlation is very weak in a fast time-varying channel, such as the vehicular channel with  $f_{d,\max} \cdot T = 0.8$  or  $f_{d,\max} \cdot T = 0.9$ , where the frequency correlation may be stronger. The frequency correlation  $\rho_f$  depends on two parameters, the first one being normalized Doppler spread  $f_{d,\max} \cdot T$  and the other one being normalized delay spread  $\frac{\tau_{\max}}{T}$ , see Eq. (18). Therefore, the performance of STBC schemes depends only on the channel's time selectivity (transmitter/receiver mobility), while the performance of SFBC scheme is related to



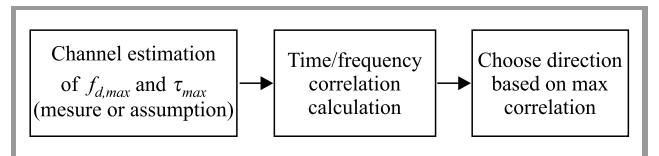
**Fig. 8.** Time frequency correlation strength in terms of  $f_d \cdot T$  ( $f_0 = 5.9$  GHz,  $T_s = 0.1$   $\mu$ s).

both time and frequency selectivity, i.e. transmitter/receiver mobility and vehicular environment.

Figure 8 shows that the channel correlation strength can be weak or strong. It is related to vehicle speed and channel delay spread. So, wireless performance may be significantly improved by adaptive switching between STBC and SFBC, based on current channel conditions. The proposed switching criterion is based on channel correlation in terms of time and frequency. Therefore, at the transmitter side, the system switches between STBC and SFBC schemes according to channel selectivity values computed based on channel correlation strength, using Eqs. (17) and (18).

The transmitter makes an estimation of time correlation strength  $\rho_t$  and frequency correlation  $\rho_f$  strength, and then decides where the correlation is stronger (in time or frequency). The vehicular transmission system only needs to estimate the maximum Doppler frequency and the channel delay spread. Specific assumptions may be made by the sender based on the maximum speed of vehicles and the maximum delay spread of the multipath channel. The proposed adaptive switching scheme is presented in Fig. 9.

Correlation strengths  $\rho_t$  and  $\rho_f$  are calculated in order to select the appropriate transmission scheme (STBC or SFBC), knowing that the channel's time selectivity depends only on the speed of the vehicle (normalized Doppler spread), while the channel's frequency selectivity depends on the speed of the vehicle and on the scatterers present in the environment (normalized Doppler spread and normalized delay spread).



**Fig. 9.** Adaptive STBC/SFBC switching in vehicular networks.



## 5. Simulation Results

In this section, theoretical considerations are verified by Matlab simulations. Performance of the proposed adaptive approach is evaluated and compared with conventional STBC and SFBC schemes. The simulations are based on the Rayleigh fading channel model and OFDM transmissions in an IEEE 802.11p physical layer.

The normalized channel model is Rayleigh, as recommended by the European Telecommunications Standards Institute (ETSI). The simulation parameters are given in Table 3.

Table 3  
Simulation parameters

|                        | Parameters         |                     | Values           |
|------------------------|--------------------|---------------------|------------------|
| Systems parameters     | System bandwidth   | $B = 10$ MHz        |                  |
|                        | Modulation         | 4 QAM               |                  |
|                        | Sampling time      | $T_s = 0.1$ $\mu$ s |                  |
|                        | OFDM subcarriers   | 52                  |                  |
|                        | OFDM guard samples | 16                  |                  |
| Rayleigh channel model | Path number        | Average power [dB]  | Normalized delay |
|                        | 1                  | -7.219              | 0                |
|                        | 2                  | -4.219              | 0.4              |
|                        | 3                  | -6.219              | 1                |
|                        | 4                  | -10.219             | 3.2              |
|                        | 5                  | -12.219             | 4.6              |
|                        | 6                  | -14.219             | 10               |

The performance is evaluated under vehicular channel conditions, and time frequency selective channels are simulated for different vehicle speeds and different channel delay spreads.

### 5.1. Performance Evaluation of Alamouti STBC Scheme

Next, sets of MISO Alamouti STBC data are used as space-time block coding on a vehicular channel. The system is simulated on both time-selective (high mobility) and time non-selective (low mobility) channels in order to evaluate their performance.

Two types of vehicular channels are evaluated. Performance of the  $2 \times 1$  STBC OFDM scheme and the OFDM single antenna scheme is compared over a flat fading channel ( $f_d = 0$  Hz, vehicle speed = 0), and the results are shown in Fig. 10. Performance of both systems ( $2 \times 1$  STBC OFDM and single antenna OFDM) in a fast fading channel ( $f_d = 500$  Hz) is shown in Fig. 11. From the figures, we may clearly see that in slow fading channels, STBC-OFDM achieves good performance compared to the single antenna system. STBC offers poorer performance, but still outperforms a single OFDM system in the case of a fast fading channel ( $f_d = 500$  Hz).

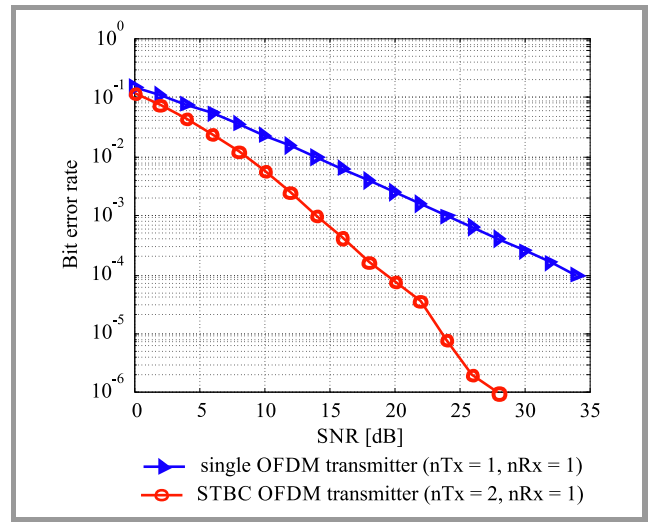


Fig. 10. Performance of single OFDM and STBC-OFDM for time invariant channel ( $f_d = 0$ ).

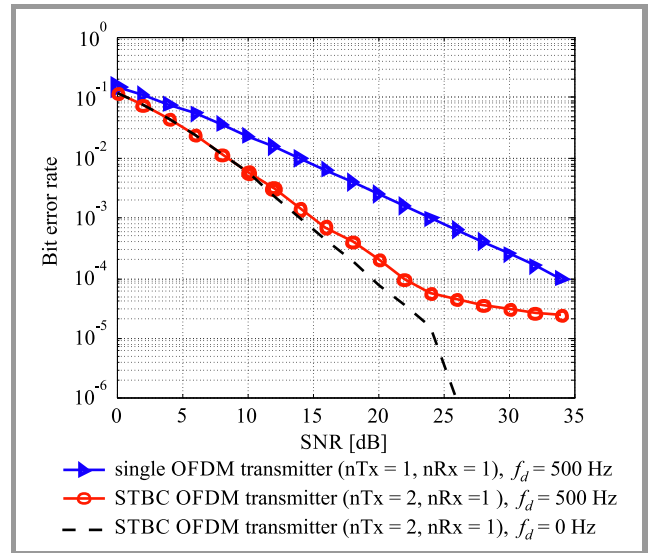


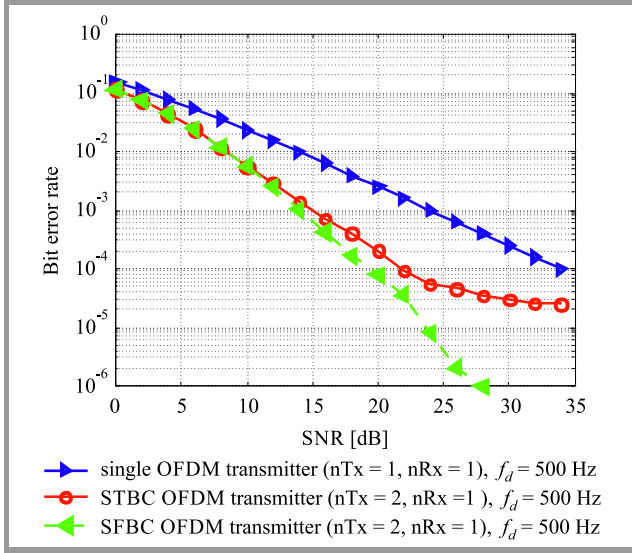
Fig. 11. Performance of single OFDM and STBC-OFDM for moderate time varying channel ( $f_d \cdot T = 500$  Hz).

### 5.2. Performance Comparison of STBC and SFBC

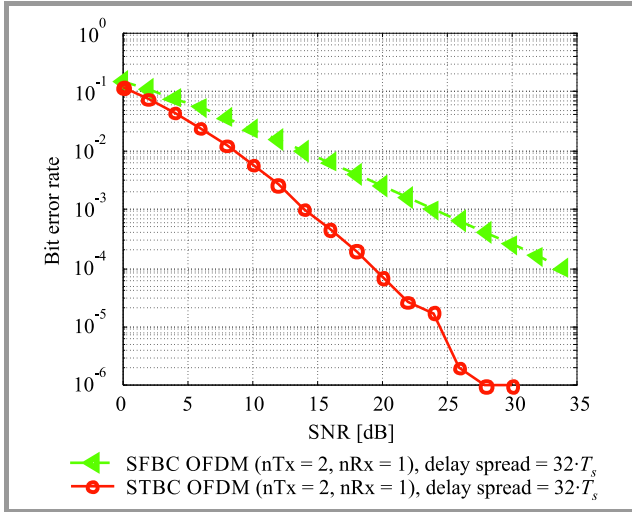
OFDM STBC and SFBC systems are simulated in order to compare the performance of the two schemes under different vehicular channel conditions. Two types of vehicular channels are simulated: the first channel is time-selective with flat frequency fading ( $f_d = 500$  Hz,  $\tau_{\max} = 0$ ), and the results are shown in Fig. 12. The other channel is frequency selective with flat time fading ( $f_d = 0$  Hz,  $\tau_{\max} = 32 \cdot T_s$ ) – the results are shown in Fig. 13.

SFBC performs better over time-selective channels, but both schemes (STBC and SFBC) achieve similar performance over time-invariant channels (Fig. 12).

Figure 13 shows that STBC performs better over a vehicular channel with severe frequency selectivity. The performance of SFBC is lost, because the assumption of  $B_c > 2\Delta f$  is not fulfilled under these channel conditions. Alamouti STBC is more sensitive to time selectivity (vehicle speed), while



**Fig. 12.** Performance comparison of STBC and SFBC for a time-selective channel ( $f_d = 500$  Hz,  $\tau_{\max} = 0$ ).

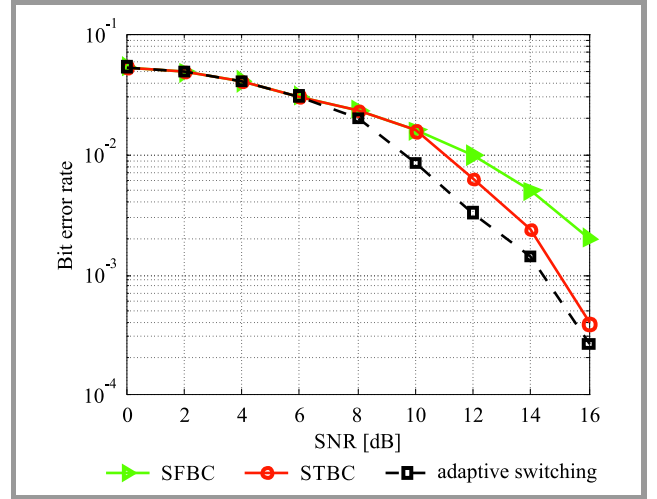


**Fig. 13.** Performance comparison of STBC and SFBC for a severely frequency-selective channel ( $f_d = 0$ ,  $\tau_{\max} = 32 \cdot T_s$ ).

SFBC is sensitive to frequency selectivity (propagation environment). The proposed switching scheme is compared with the two conventional STBC and SFBC schemes.

### 5.3. Performance Evaluation of The Proposed Adaptive Switching Method

In this section, a time-frequency selective channel is generated by varying the vehicle speed and the propagation environment. The channel's Doppler frequency is randomly generated between 0 and 500 Hz in order to simulate real vehicular channel conditions. Similarly, channel delay spread is randomly generated between 0 and  $32T_s$ . The performance is illustrated in Fig. 14. The bit error rate (BER) of the proposed switching design remains below that of STBC and SFBC schemes. As expected based on theoretical analysis, it is shown that the proposed design is better than STBC and SFBC schemes performing solo in



**Fig. 14.** Adaptive switching performance over time frequency selective channel.

vehicular channels with varying time-frequency selectivity levels. The proposed technique may be extended to MIMO  $2 \times 2$  or massive MIMO configurations.

## 6. Conclusion

The proposed adaptive switching technique improves the performance of vehicular communications over time-frequency selective channels. The combination of MISO STBC/SFBC with OFDM for the IEEE 802.11p standard is capable of satisfying new requirements concerning vehicular communications. It has been found that the SFBC design outperforms STBC under high mobility environments. However, the SFBC scheme offers poor performance in severely frequency-selective channels. Based on the previous results, an adaptive switching method is proposed, improving the performance of conventional STBC and SFBC schemes in vehicular channels.

From the simulation results concerning single antenna IEEE 802.11p systems and upgraded systems with MISO STBC/SFBC adaptive switching deployed, it may be concluded that the MISO system requires lower transmission power to achieve the same BER as a single antenna system, simultaneously offering higher data rate communications and increasing system reliability. Furthermore, the adaptive design proposed in this paper offers good performance when deployed in dynamic vehicular channels.

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# Fuzzy-based Description of Computational Complexity of Central Nervous Systems

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<https://doi.org/10.26636/jtit.2020.145620>

**Abstract**—Computational intelligence algorithms are currently capable of dealing with simple cognitive processes, but still remain inefficient compared with the human brain's ability to learn from few exemplars or to analyze problems that have not been defined in an explicit manner. Generalization and decision-making processes typically require an uncertainty model that is applied to the decision options while relying on the probability approach. Thus, models of such cognitive functions usually interact with reinforcement-based learning to simplify complex problems. Decision-makers are needed to choose from the decision options that are available, in order to ensure that the decision-makers' choices are rational. They maximize the subjective overall utility expected, given by the outcomes in different states and weighted with subjective beliefs about the occurrence of those states. Beliefs are captured by probabilities and new information is incorporated using the Bayes' law. Fuzzy-based models described in this paper propose a different – they may serve as a point of departure for a family of novel methods enabling more effective and neurobiologically reliable brain simulation that is based on fuzzy logic techniques and that turns out to be useful in both basic and applied sciences. The approach presented provides a valuable insight into understanding the aforementioned processes, doing that in a descriptive, fuzzy-based manner, without presenting a complex analysis.

**Keywords**—*cognitive deficit, cognitive function, computational simulation, fuzzy descriptors.*

## 1. Introduction

One of the biggest challenges faced by contemporary science is concerned with the problem of how external stimuli are gathered, integrated, analyzed and interpreted within the human brain's neural connections, and the manner in which they are then reflected by human behavior (even as a simple response to stimuli). Scientists continuously search for effective algorithms allowing to code/decode information to/from neural tissue. The so-called human connectome may reflect some of the aforementioned information, also that regarding memory and feelings. Brain mapping (reflected in the connectome) still remains the very first step in the process of obtaining the knowledge required. There is a need for a deeper understanding of built-in

memories and feelings. The scale of “thought” processes may be much larger than that of processes taking place within neural networks only. Even a simple analysis of a sentence requires more than activity of the neural networks only (that is similar in many healthy people). It also calls for emotional meaning, social context and many other aspects, thus requiring an advanced and sophisticated analysis of numerous diverse factors that are not always self-explanatory. Hence, there may be a systematic association between sentence meaning and neural network activation features. Experiments such as “Initiative 2015” may allow to copy the structure of a human brain into a computational avatar.

Rapid biological research developments that continue to be observed since the 1990s have brought about a considerably increase in the amount of biological data of various types. The ability to perform a computational analysis of the aforementioned data sets has given rise to a belief that it will be possible to reflect the full understanding of nature and functionalities of living organisms, and especially of the human central nervous system (CNS). Computational models are regarded as scientific tools linking the commonly known hypotheses and outcomes of experimental studies. Such an approach allows to analyze, in great detail, the building blocks of the human body. Unfortunately, not that much attention is paid to interactions between them, despite the fact that we are aware that the aforementioned interactions may play crucial roles, for instance in more cognitive functions. Such building blocks of the human brain are usually interconnected by a very dense and complex network of bidirectional interactions. The assumption that the structure of this network may determine some basic features of human brain's functioning may be true. Despite the development of novel techniques, such as functional magnetic resonance imaging (fMRI) or positron emission tomography (PET), the current state of our knowledge may be strongly limited by the restricted scope of analyses performed using the existing tools which are poorly suited to cope with the interaction networks referred to above. There is a need for further adaptation of computational methods to the specific nature of neurobiological systems that are dealt with by biology of neurosystems [1].

### 1.1. Scope of the Paper

This article presents a short description of computational complexity levels and uncertainty of information, based on an example of degenerative changes and their neural correlates. The concept of their computational simulation is described as well, and the associated discussion is presented. First, a review of current knowledge concerning the investigated group of disorders and computational approaches to associated neurodegenerative changes analysis is presented, as it may help develop new fuzzy-based therapeutic strategies tailored to the profile of a specific patient. Next, the paper proposes a consistent computational method for describing and modeling such a complexity. The authors aim to offer a coherent concept focusing on cost efficiency. The precise methods that are available currently are expensive and time-consuming. Furthermore, connectivity computed with the use of different methods may differ significantly. Despite the efforts taken, results obtained with the use of diffusion tensors, spherical deconvolution, ball-and-stick models, persistent angular structures, etc., may not be the same as those of deterministic and probabilistic tractography algorithms relying on human datasets. This may be caused by the fact that connectivity may be presented in many ways, e.g. as matrices, connectograms. Fiber crossing models are able to reveal connections between more brain areas than a simple tensor model, and probabilistic approaches show more connected regions but lower connectivity values than deterministic methods.

Section 2 presents a review of current state of knowledge in the field that this paper is concerned with. Section 3 introduces computational complexity of brain description, based on the case of Alzheimer's disease. Section 4 presents the fuzzy-based proposal for computations and description brain elements. It is followed by a discussion and, finally, conclusions are presented in Section 6 of the paper.

## 2. Computational Complexity

Generalization and decision-making typically require an uncertainty model related to the decision options and using tools based on the probability theory. Models of such cognitive functions usually interact with reinforcement learning in order to simplify complex problems. Decision-makers are assumed to choose from the available decision options to ensure that their choices are rational. They maximize the subjective, overall utility expected, given by outcomes in different states, weighted with subjective beliefs about the occurrence of those states. Beliefs are captured by probabilities and new information is incorporated using the Bayes law.

Fuzzy-based models propose a different approach – they may give rise to a family of novel methods for a more effective and neurobiologically reliable brain simulation based on fuzzy logic techniques. They may be useful in such complex situations as moral principles guiding social decision-making (the so-called moral strategy), even if different participants spontaneously and consistently employ

different moral strategies (guilt aversion, inequity aversion, moral opportunism) in situations in which people adaptively switch between them.

Anatomical connectivity (AC), as an example of structural connectivity, reflects the connectome through synaptic contacts between neighboring neurons or fiber tracks connecting neuron pools in spatially distant brain regions. Functional connectivity (FC) reflects the temporal dependency of neuronal activation patterns of anatomically separated brain regions, i.e. statistical dependencies between distinct and distant regions of information processing neuronal populations. FC refers to interregional synchrony of low-frequency fluctuations, where *low* denotes frequencies of  $\nu \leq 0.1$  Hz. The term *synchronize* refers to generalized synchronization which is defined through the mapping:

$$\Psi : S- > R, \quad (1)$$

where  $S$  is the seed phase space,  $R$  is the response phase space:

$$a_i(t) = \Psi[a_j(t)], \quad (2)$$

with some functional  $\Psi$  and related seed  $a_s(t)$  and response  $a_r(t)$  activities, where  $i = s$ , and  $j = r$  for subsequent states  $i, j$  across the spaces mentioned above [2], [3].

There are two basic computational methods determining whether other regions are functionally connected to these seeds based on:

- cross-correlation analysis (CCA) between the BOLD time courses of the seed region and any other brain region, measured by the Pearson correlation coefficient  $\rho_{qs}$  such as:

$$\begin{aligned} \rho_{qs}(\tau) &= \frac{\sigma_{qs}(\tau)}{\sqrt{\sigma_q \cdot \sigma_s}} = \\ &= \frac{\langle (a_q(t+\tau) - \langle a_q(t) \rangle)(a_s(t) - \langle a_s(t) \rangle) \rangle}{\sqrt{(\langle (a_q(t+\tau) - \langle a_q(t) \rangle)^2 \rangle \cdot \langle (a_s(t) - \langle a_s(t) \rangle)^2 \rangle)}, \end{aligned} \quad (3)$$

where  $\tau$  – predefined time lag  $\sigma_i$  – variance of regional activity in the query region ( $i = q$ ) or seed region ( $i = s$ )  $\sigma_{qs}(\tau) = (a_q(t+\tau) - a_q(t))(a_s(t) - a_s(t))$  – covariance of the fluctuations in neuronal activity in the query and seed regions, respectively,

- coherence in the frequency domain:

$$H_{qs}(\nu) = \frac{|S_{qs}(\nu)|^2}{S_{q,q}(\nu) \cdot S_{s,s}(\nu)}, \quad (4)$$

where:  $S_{ij}(\nu)$  is the Fourier cross-spectrum ( $i = q, j = s$ ) or the Fourier power spectrum ( $i = j = q, s$ ) of the related covariance functions,

- coherence in the frequency domain [3]:

$$H_{qs}(\nu) = \frac{|S_{qs}(\nu)|^2}{S_{q,q}(\nu) \cdot S_{s,s}(\nu)}. \quad (5)$$

Determination of decomposition and clustering requires the application of one of the following techniques.



The first one is singular value decomposition (SVD) [3]:

$$X = U\Sigma V^T, \quad (6)$$

where:  $X : N \times M$  dimensional matrix of zero mean data with all  $M$  functional images concatenated into  $M$  column vectors containing  $N \cdot M$  pixels each,  $U : N \times N$  dimensional matrix of eigen images of the  $N \times N$ -dimensional correlation matrix,  $V : M \times M$  dimensional matrix of eigenvectors of the corresponding kernel matrix  $K = X^T X$ ,  $\Sigma$  : rectangular  $N \times M$ -dimensional matrix containing non-negative, real-valued singular values along its diagonal with only  $\min(M, N)$  singular values being different from zero.

The next two methods are principal component analysis (PCA):

$$XX^T = U\Sigma\Sigma^T U^T = \tilde{U}\tilde{U}^T, \quad (7)$$

and independent component analysis (ICA):

$$X^T = MH, \quad (8)$$

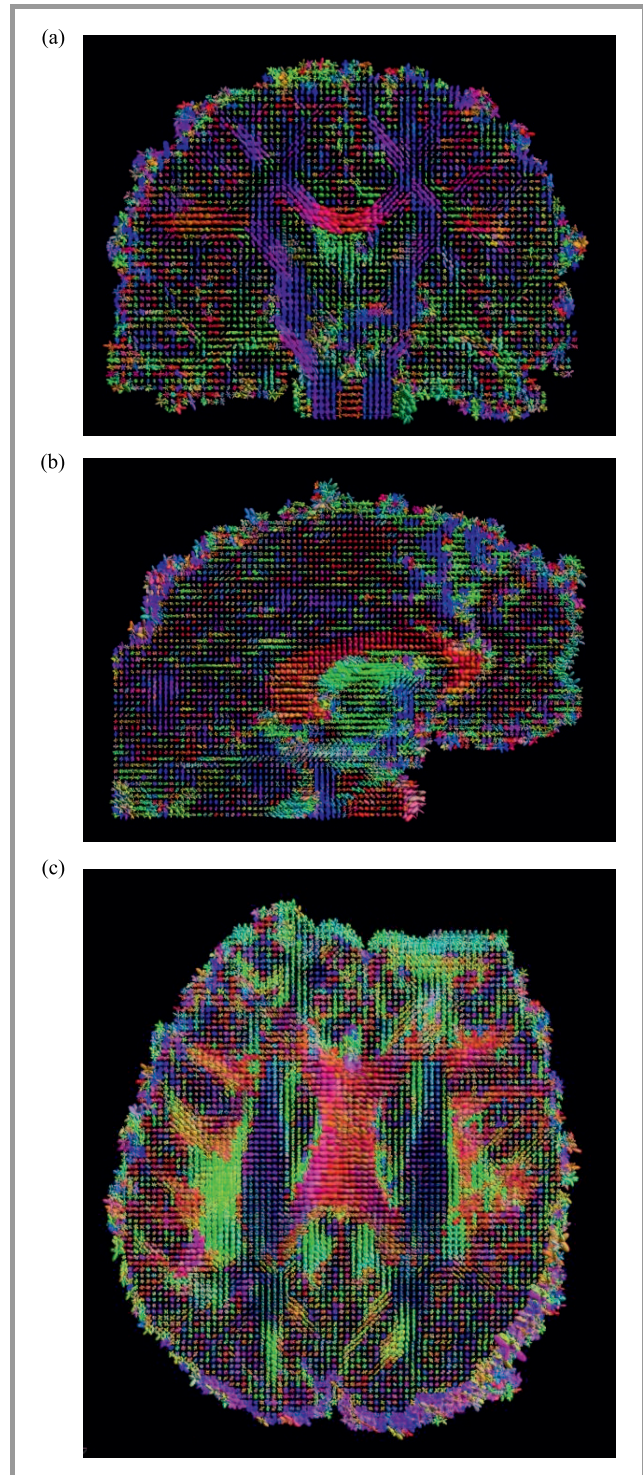
where:  $X^T$ : matrix containing spatial activity distribution in its rows and different observation time points in its columns,  $H$ : matrix containing, in its rows, independent spatial activity distributions which are assumed to best characterize the observations,  $M$ : containing in its rows the corresponding weights with which each independent component contributes to the observation at any given time point.

The last technique is non-negative matrix and tensor factorization (NMF/NTF) [3].

An alternative approach consists in the application of multivariate auto-regressive models (MVAR) [4], [3]. Effective connectivity (EC) reflects the influence that one neuronal system exerts upon another, thus reflecting causal interactions between activated brain areas, i.e. combines structural and effective connectivity [3].

Structural connectivity has been studied by modeling statistical dependence between the features of cortical regions. The main statistical metrics used include correlations between gray matter thickness. In addition, it has been proposed that partial correlations be used for inferring brain structural connectivity using gray matter volume, instead of the Pearson correlation, since this approach removes the effects of confounding correlations introduced due to canonical dependence between data (Figs. 1–3). Three basic factors play a key role here: methods of parcellation of the cortex (i.e. node selection criteria), threshold selection (i.e. for connection metrics), strength of the relationship between anatomical structure and cognitive function (both in healthy and damaged nervous systems).

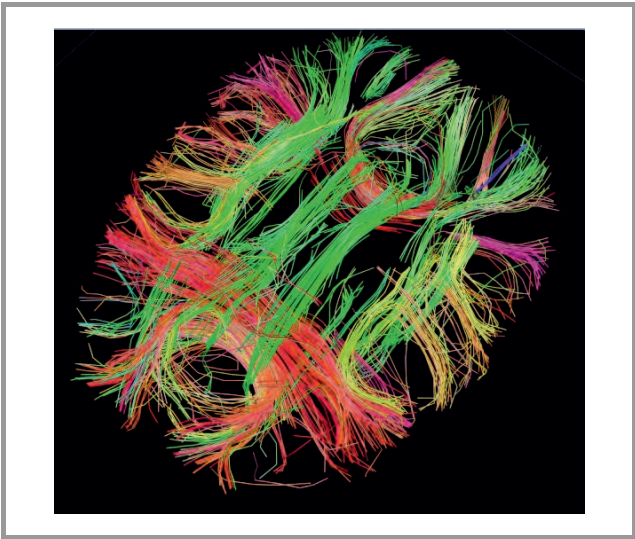
Further on in the text, a proprietary approach to the Alzheimer's disease (AD) is proposed. AD is the most important age-related neurodegenerative disorder in developed societies. Prevalence of AD in Europe was estimated at 5.05%. Prevalence in women is higher (7.13%) compared to 3.3% in men, and increases with age. Incidence of AD in Europe was 11.08 per 1000 person-years. Incidence in women is 13.25 per 1000 person-years men, compared to 7.02 per 1000 person-years in men, and increases with age.



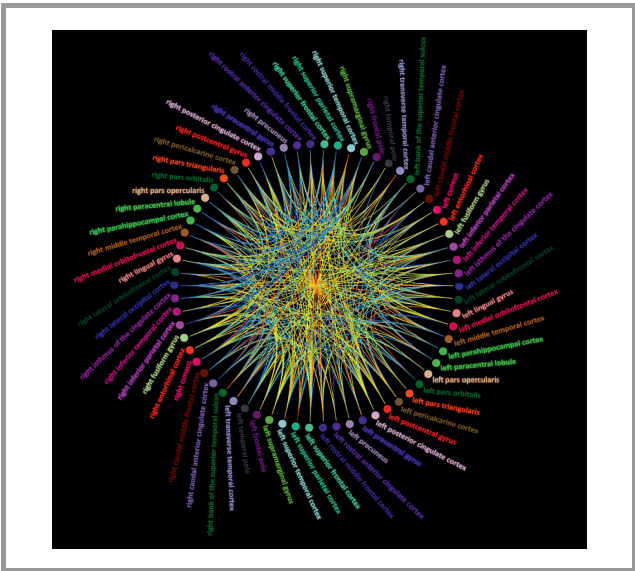
**Fig. 1.** High-angular resolution diffusion imaging of the human brain: (a) coronal view, (b) sagittal view, (c) axial view; red color describes left-right fibers, green shows anterior-posterior fibers, and blue indicates fibers through the brainstem [5]. (For color pictures see the electronic version of the paper).

Currently, AD diagnosis is based on a subtle cognitive decline in the patient's preclinical health status, detected using neuropsychological tests. Treatment is based on symptomatic therapies, and research enhancing early detection of the disease and preventing pathological changes in the





**Fig. 2.** White matter fiber architecture of the brain; red color shows left-right fibers, green presents anterior-posterior fibers, and blue color illustrates fibers through the brainstem [5].



**Fig. 3.** Architecture of complex interconnections of the brain; warmer colors identify stronger connections and cooler colors identify weaker connections [5].

brain is of key importance. Computational models may significantly increase the understanding of the mechanism underlying the onset of neurodegenerative diseases, and may support development of novel diagnostic and therapeutic methods. Only several studies published over the past decade have been identified line [6]–[12].

### 3. Alzheimer Disease Case

#### 3.1. aMCI and Late Life Depression as AD Predictors

AD is one of the most prevalent neurogenerative brain disorders, affecting an increasing number of people in

aging societies of developed and developing countries and creating a number of medical, social and economic challenges. Current research focuses on the following key issues: increasing number of people diagnosed with AD, development of effective therapies and decreasing costs associated with treating and monitoring patients. The diagnostic criteria remain ambiguous, making it difficult to recognize AD symptoms as early as possible. The amnesic form of mild cognitive impairment (aMCI) is regarded as a precursory phase of AD. Even 75–80% of patients with aMCI finally develop AD [13].

Cognitive functions deficits are often accompanied or preceded by emotional, behavioral and motivational disorders. Late life depression occurs in up to 20% of patients with AD and affects about 3.0–4.5% of adults aged 65 and over [14], [15]. Both aMCI and late life depression are perceived to be risk factors for the onset of dementia in elderly people, being also significant early markers thereof. Elderly people experience health problems that are accompanied by dementia and often also by depression. By taking into account late age depressive disorders in determining the pathogenesis of neurodegenerative processes, another step towards their better understanding may be taken. Research concerning relationships between dementia and depression, their causes, clinical manifestations and treatment is of significant clinical importance.

AD constitutes a dominant form of dementia in Europe (up to 60% of all cases). The exact percentage share of patients with late life depression converting into aMCI or dementia is still unknown, especially taking into consideration scarce knowledge about the preventive influence of antidepressants in the deterioration of cognitive functions. A close dependency may be observed here: occurrence of dementia impacts the psycho-pathological picture of depression and, vice versa, depression may dramatically worsen the health status of elderly patients. There is no doubt that aMCI patients display more severe cognitive impairments affecting all domains, while late life depression individuals show worse executive function- and memory-related results [16]. Unfortunately, no validated markers are capable of precisely predicting the conversion from aMCI to AD. Current clinical diagnosis is based mainly on observation and neuropsychological examination, as well as on AD biomarkers:  $\beta$ -amyloid in cerebrospinal fluid, use of PET and assessment of neuron damage with the use of tau marking, assessment of brain metabolism in PET and cerebral atrophy in MRI [17].

#### 3.2. Neural Correlates of aMCI and Late Life Depression

Structure and function of the central nervous system (CNS) of patients with (a)MCI, depression and AD differ from those of healthy elderly individuals [18], [19]. However, exact differences within brain structures and functions between these MCI subtypes, depression, and AD have not been studied yet. Neuroimaging as a way of determining the conversion of aMCI patients with apolipoprotein E4

(ApoE-E4, aMCI-E4 patients) to AD was researched by Chen *et al.* [20]. Impaired structure and function of the hippocampus, constituting a part of the medial temporal lobe memory system, is regarded as a valuable predictor of progression from aMCI to AD. As it is affected by neuropathological changes at the earliest phase, it may show progressive degeneration associated with progression from aMCI to AD [20]. Increased diagnostic efficiency may show a combination of biomarkers derived from neuroimaging (especially MRI), ApoE, and cognitive scores [21].

Hippocampus, posterior cingulate cortex and corpus callosum are currently the key areas of studies focusing on aMCI and AD that are based on the use of structural MRI (sMRI), functional MRI (fMRI), and diffusion tensor imaging (DTI). The neuroimaging abnormalities observed during the progression of aMCI to AD may affect, primarily, posterior brain regions and may gradually progress to the anterior areas. Current results of the aforementioned studies, however, are insufficient to build effective predictive models that could be applied in screening and diagnosing aMCI and AD. As far as the assessment of the specific risk of conversion from aMCI to AD is concerned, the Brain Age Gap Estimation (BrainAGE) method, determining the impact of ApoE genotype on structural brain aging, proves to be an accurate tool, even if information on the patient's ApoE status is missing [21]. A multi-tool approach (sMRI, fMRI, DTI) has been adopted by Yu *et al.* in [23] to analyze neuroimages of brain in patients with aMCI and vascular MCI (VaMCI). Microstructural alterations revealed by multimodal MRIs, including sMRI, resting-state functional MRI and DTI, and accompanied by neuropsychological changes, were observed in [24]. The results of recent EEG-based studies focusing on aMCI, late life depression and AD, as well as on the diagnostic value of spectral analysis of EEG (at rest) in the process of differentiating aMCI in routine clinical practice, are questioned. Analysis of synchronicity of bioelectrical activity of the brain may improve the situation. Combination of neurophysiological and neuroimaging methods may provide much better outcomes.

### 3.3. Computational Concepts of aMCI Simulation

The correlation between cognitive states and network characteristics is rather clear, but remains multi-factorial. From the computational modeling point of view, AD has been previously linked with the concept of underlying pathological brain network disorganization and less efficient information processing.

The current approach has not provided explanations for pathological alterations of brain network functions during the preclinical phase of aMCI yet. The following types of functional disorganization was shown:

- functional disorganization correlated with the participant's generic cognitive status in healthy people, in patients with aMCI and with mild dementia,

- “small world” brain architecture identified in aMCI patients relative to patients with mild dementia,
- significant reduction in brain architecture in both aMCI and MD patients, relative to the group of healthy controls.

Pathological changes in the organization of brain networks are caused mainly by deficient local information processing (lower local clustering, reduced strength of functional hubs), despite some compensatory phenomena observed and having the form of the formation of additional hubs in left frontal and parietal regions. The aforementioned changes resulted in declined functional organization of the network, even during the prodromal/preclinical phase [23]. On the other hand, induction of neuroplasticity of the mature human brain by combining cognitive and physical activity (e.g. dancing) leads to the prevention of dementia. Traditional Greek dances may improve optimal network performance (small-world properties, local network changes causing better information flow and functional reorganization of the network nodes) [25]. Moreover, there may be diverse computational mechanisms of aMCI, since various aMCI clinical sub-types have been identified as being of the single domain (SD) or multi-domain (MD) variety, resulting in the heterogeneous syndrome with different probabilities of progression from aMCI to AD. Abnormalities underlying MD-aMCI distributed across temporal, frontal, and parietal cortices showed that MD-aMCI is a transition stage between SD- aMCI and AD [26].

The brain network in aMCI patients displayed a disconnection syndrome and a loss of small world architecture. The aforementioned features may be influenced by numerous factors – see e.g. deterioration of the cognitive state in diabetic patients, reflecting a less optimal network organization [27]. Cortical sources small worlds net of resting EEG in eyes-closed/open differs in aMCI and AD patients in comparison to healthy subjects. The aMCI small world architecture presents midway properties between AD and healthy subjects, but in the low frequencies (delta and theta bands), the aMCI group presents a network architecture that is similar to that of healthy individuals. Such results may confirm the hypothesis stating that aMCI constitutes an intermediate step in the progression of AD [28].

Small world topology networks constructed by cortical mean diffusivity in aMCI and AD are not well known so far. But one may state that patients with aMCI and AD show abnormal small-world properties of cortical diffusivity networks reflected by a higher degree of clustering and longer path lengths, i.e. less optimal topological organization. According to the best current knowledge, changes of cortical diffusivity in topological network organization, the average complexity of connections and the disrupted hub regions in aMCI may be useful in identifying patients in the prodromal stage of AD, as they reflect microstructural neurodegeneration [29]. Network construction optimization problems may be a key for a deeper understanding of the

complexity of the transition between healthy, aMCI, and AD states [30]–[32].

Our approach may be simpler than a complete solution to the aforementioned problem. The complete solution requires that two areas be converged:

- underlying mathematical model of the partial differential equation (PDE, e.g. Bloch-Torrey PDE), harmonic analysis, and algebraic geometry,
- technology, primarily diffusion magnetic resonance imaging (dMRI).

The aforementioned approach creates an efficient solution to the 3-dimensional imaging problem. It is achieved firstly by the reconstruction of a homogeneous polynomial representing a real-valued function on a sphere, based on dMRI data. Then, these data sets are analyzed by being decomposed into a sum of powers of linear forms. The ultimate result requires combination and mapping of neural fibers, i.e. multiple-fiber reconstruction of the brain map. Multiple compartment transformation of the Bloch-Torrey PDE is used to reduce oscillations in the searched-for solution and simplifies the implementation of the boundary conditions:

$$\frac{\delta}{\delta t} = -I\gamma f(t)G(r)M(r,t) + \nabla \cdot (D(r)\nabla M(r,t)),$$

$$r \in \bigcup \Omega^I, \quad (9)$$

and for each compartment  $\Omega^I$ :

$$\begin{aligned} \frac{\delta}{\delta t} \int_{\Omega^I} M v dr &= -I\gamma f(t) \int_{\Omega^I} G M v dr \\ &- \int_{\Omega^I} D \nabla M \cdot \nabla v dr + \int_{\delta \Omega^I \cap \Gamma} D \nabla M \cdot n^I v ds \\ &+ \int_{\delta \Omega^I \cap \delta C} D \nabla M \cdot n^I v ds. \end{aligned} \quad (10)$$

Then, the similarity, weak form of the steady-state Laplace PDE is:

$$\begin{aligned} \int_{\Omega^I} D \nabla W \cdot \nabla v dr - \int_{\delta \Omega^I \cap \Gamma} D \nabla W \cdot n^I v ds \\ + \int_{\delta \Omega^I \cap \delta C} D \nabla W \cdot n^I v ds = 0, \end{aligned} \quad (11)$$

where:  $\gamma = 2.67513 \cdot 10^8 \text{rads}^{-1} T^{-1}$ ,  $D(r)$  – intrinsic diffusion tensor,  $n^I$  – outward pointing normal,  $v$  – test function,  $M(r, t)$  – magnetization, function of position  $r$  and time  $t$ , depends on diffusion gradient vector  $g$  and time profile  $f(t)$  [33].

EniCS finite elements platform and the Salome mesh generator may be applied to solve the aforementioned problems numerically.

As far as less efficient information processing is concerned, brain network dysfunction analyzed at the systemic level may be a key issue: aMCI with AD showed abnormali-

ties both in regional brain activation and large-scale brain networks:

- aMCI patients show hypoactivation in default, visual, and frontoparietal networks, relative to healthy controls,
- AD patients show hypoactivation in default, visual, and ventral attention networks, relative to healthy controls,
- both group of patients (aMCI and AD) show hyperactivation in default, frontoparietal, ventral attention, and somatomotor networks, relative to healthy controls,
- MCI and AD present different pathological changes, sharing even similar compensatory large-scale networks relied upon in fulfilling cognitive tasks [32].

The response time (RT) necessary for safe, appropriate, and efficient environmental interaction required in real life is normally distributed for cognitively healthy older adults, but not for the aMCI group [22]. A high degree of intra-individual variability (IIV) has been observed, and results reveal a dichotomy between the preservation of RT along with aging, a phenomenon that is probably strongly related to the research methodology used. Current approaches are based on simple clinical tests and scales - there is a lack of a widely accepted integrated computational approach.

The first of the aforementioned approaches to brain network disorganization gives a detailed picture of brain connections, but not so of brain mechanisms, especially mechanisms of a higher level of abstraction associated with memory, reasoning and decision-making processes. The other approach (less efficient information processing) shows the behavior and, partly, the associated mechanisms, but requires that the computational description rules reflecting its function be determined.

## 4. Fuzzy-based Approach

Our approach to fuzzy-based computational analysis of brain processes has already been described before [34], [35]. We developed a novel computational tool: hierarchical fuzzy systems based on ordered fuzzy numbers, i.e. the direction of the calculation/operation has its own meaning [36], [37]. Thus, the theory of fuzzy sets offers help in defining formal models in situations in which only a linguistic description is possible. It may be easily adapted to reflect the “fuzziness” of neurodegenerative changes associated with AD and aMCI. A fuzzy description of signal transformation performed through the complex spatial properties of brain networks, from optimization of network structure thanks to long time developmental and learning processes, toward a non-optimized or even distorted connectome due to neurodegenerative processes, and with time complexity of the step-by-step transition between healthy brain, through aMCI, toward AD,

allows for more precise control and prediction. The modular fuzzy approach proposed better reflects various speeds and other properties of disturbed simultaneous hierarchical data processing, as well as co-occurrence of numerous signals. The models are more flexible and simpler to adapt. Thanks to the aforementioned properties, low-cost less time-consuming simulations, a considerably higher number of variables may be tested, enabling to reflect IIV. Intuitive information provided by experienced brain modeling scientists may be represented in a simpler manner, with growing and decreasing dependencies [38]. Invalidly synchronized (or non-synchronized) structures may be described, for instance by weaker-than-usual co-occurrence of excitation/inhibition, making them hard to detect. Such an approach may help reflect structural connectivity (SC) as a part of functional connectivity (FC).

The aforementioned approach may introduce new quality to various cognitive changes and their differential diagnosis. Changes in the patient's nervous networks may take place much earlier than visible symptoms and diagnostic outcomes may be identified. Therefore, the proposed approach simultaneously creates a need for novel brain state visualization tools. Furthermore, the proposed fuzzy-based approach allows for more advanced OFN-based brain function trend modeling. Such an approach may offer an increased predictive value based on current and new knowledge gathered.

A proper choice of fuzzification and defuzzification method exerts a key impact on the correctness and effectiveness of the entire model, including accumulation of losing bits of information (see [39]). Our proposal processes a complex functionality that is expressed linguistically by a fuzzy system, without fuzzification and defuzzification stages used as a fuzzy inference block (FIB). Moreover, its conceptual basis is enhanced by cascade-based modeling of complex relations, described linguistically. Every FIB may represent one agent from the multiagent architecture. Despite the high degree of complexity of the model, we can still describe relations at various stages linguistically. Multi-input and multi-output semi-cascades of FIBs may simultaneously reflect various conditions and co-operation states of diverse brain networks, with the required scalability. Hence, it seems that the small world network is widely accepted as a scalable intrinsic miniature of the nervous system.

The maximum value of the reward function  $R$  guarantees that the best change may be found in a given step. The reward function changes dynamically during the time steps. Therefore, better choice is improving other parameters e.g. learning rate coefficient. The change of the weight between  $n_i$  and  $n_j$  is calculated as:

$$\frac{dw_{ij}}{dt} = aR(t)e_{ij}(t), \quad (12)$$

where:  $a$  – learning rate,  $R$  – reward function,  $e_{ij}$  – eligibility trace between  $n_i$  and  $n_j$  neurons from the two adjacent layers.

In our approach, we focus *inter alia* on modeling the learning rate with OFN, and on trend analysis. Such an approach may reflect the stochastic behavior of simple, reliable, very fast, low-power computing devices embedded in recursive brain-like architectures.

## 5. Discussion

Computational neuroscience is developing rapidly and provides a variety of modern tools for analyzing functional interactions of the brain and for detecting pathological deviations caused by neurodegeneration. Identification of functional and neural markers enabling to diagnose adverse cognitive changes associated with MCI and depression, and leading to an early diagnosis of AD, is a key objective of modern neuroscience and geriatrics. Unfortunately, there is lack of comparable reviews. What more there is hard to cover all possible issues and factors in a single study.

The point of view of the authors formulates a concept that is still at an early stage of development and needs further discussions. It aims at ensuring lower computational complexity, as well as wide acceptance and availability, but still suffers from many limitations that are mentioned below. Further tests and numerical results, including compartmental studies, will be presented in the future. However, an advanced approach based on computational intelligence seems to constitute another breakthrough in brain function modeling.

Limitations of the proposed concept are twofold: they result from the complexity of the brain (and lack of its direct modeling) and from the characteristics of the proposed concept. The diversified location of brain changes, the level neural correlations between aMCI and depression, the connection between distant areas of CNS, diverse clinical subtypes, and a few clear "cause-effect" relationships are the main challenges that need to be faced while conducting research. There is a need for novel approaches which meet all of the abovementioned criteria, simultaneously being more complex and precise, offering enough explanatory and predictive power, as well as allowing for further development of computational models of neurologic diseases and deficits in elderly patients [40], [41], [42], [43].

Computer tools allow to collect and analyze (with the use of artificial intelligence) significant amounts of information. Diverse combinations, such as AD only, AD and coexisting depression, AD with aMCI, AD with aMCI and associated depression, or AD with depression, may be identified in an easier manner. The area and progress of pathological CNS processes should be defined by means of a certain boundary point, thus enabling them to be distinguished precisely.

A comprehensive patient diagnosis is needed to get a clear understanding of the impact of various areas, levels and severities of underlying neurodegenerative changes. The main advantages will have the form of a deeper understanding of dementia's development pathomechanisms. The ulti-

mate result will be a flexible, multidimensional diagnostic procedure, enabling to early detect pre-AD effects and to apply a suitable treatment wherever possible.

### 5.1. Directions for Further Research

Future research should focus primarily on developing a software tool, but there are many other issues that may contribute to a rapid increase in the potential of the proposed approach. A fuzzy system may be suitable for shape the control signal and for developing much more advanced control structures. Fuzzy logic and model predictive control methodologies may create a novel multi-parametric programming (MPT) approach. Advanced BCI applications require real time solutions, but brain subsystems simulation requires only low-cost post-hoc operation. Therefore, the use of the fuzzy MPT technique may be an efficient and a relatively simple and cheap solution [44].

Despite the great expressive power of fuzzy systems, they also suffer from specific limitations in terms of modeling neurobiological systems. Determination of exact values of some parameters and of the associated structures is not always possible. Therefore, it is difficult to exactly correspond to some quantitative properties of the modeled system. Even a formal analysis of graph-based models, such as Petri nets (structure of a directed bipartite graph), may be general. We should be also aware that the lead time of neurobiological responses exerts a significant influence on the overall behavior of the human body and on the modeled system. Hence, Petri nets may enable, with time, to perform detailed and accurate modeling and analysis of the modeled system [1].

There are also other concepts that should be taken into consideration in the course of future studies, such as granular computing [45], [46] or flexible and reliable fuzzy logic-based decision support systems [47].

## 6. Conclusions

In this paper, the concept of a fuzzy logic-assisted decision support system (DSS) has been introduced. Thanks to FL, the proposed solution provides a valuable insight into the understanding of the aforementioned processes, relying on descriptive, fuzzy-based methods, without the need to perform a complex analysis. Early and precise identification of patients with aMCI, late life depression or AD is of great significance for introducing successful prevention and intervention programs. Future studies on aMCI and late life depression should cover computational models and ought to provide stronger evidence allowing to adopt a novel approach to prevention, diagnosis and treatment of AD. Such an approach may fill the existing gap between theoretical hypotheses and experimental studies. The majority of components of the aforementioned mechanisms and associated treatment strategies still need to be confirmed by future studies.

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# Application of Graph Theory Algorithms in Non-disjoint Functional Decomposition of Specific Boolean Functions

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<https://doi.org/10.26636/jtit.2020.142520>

**Abstract**—Functional decomposition is a technique that allows to minimize Boolean functions that cannot be optimally minimized using other methods, such as variable reduction and linear decomposition. A heuristic method for finding non-disjoint decomposition has been proposed lately. In this paper, we examine how the usage of different graph theory techniques affects the computation time and the quality of the solution obtained. In total, six different approaches were analyzed. The results presented herein prove the advantages of the proposed approaches, showing that results obtained for standard benchmark M-out-of-20 functions are better than those presented in previous publication. Results obtained for randomly generated functions prove that time complexity and scalability are significantly better when using the heuristic graph coloring algorithm. However, quality of the solution is worse, in general.

**Keywords**—logic synthesis, functional decomposition, non-disjoint decomposition, index generation functions.

## 1. Introduction

The growing use of Field Programmable Gate Arrays (FPGAs) has led to a significant increase in interest in efficient Boolean function minimization methods. Specific functions known as index generation functions (IGFs) have been gaining in popularity among researchers. Several examples of IGFs applications are presented in the literature [1], e.g. in connection with IP address tables, terminal access controllers and computer virus scanning circuits. Those functions represent the following mapping:

$$F : D^N \rightarrow \{1, 2, \dots, K\}, \quad (1)$$

where  $D^N$  is a set of  $K$  different binary  $N$ -bit vectors, called registered vectors. The function maps a unique integer value to each of them. If the input does not match any of the registered vectors, the function produces a zero value. The important property of index generation functions is that those functions are not fully defined. Thus, logic synthesis algorithms may be used to find a representation of the function using a number of variables that is lower than  $N$ . In particular, the optimum number of variables is:

$$\kappa = \lceil \log_2(K + 1) \rceil. \quad (2)$$

Scientists focus mostly on two minimization techniques, i.e. reduction of variables and linear decomposition. However, those methods do not provide an optimized representation for all functions [1], [2]. Therefore, new solutions and papers concerned with these issues are emerging. Recent studies [3]–[6] investigate the application of functional decomposition in minimization of index generation functions. Functional decomposition [7] is a method, in which a Boolean function  $F(X)$  with numerous input variables  $X = \{x_1, x_2, \dots, x_n\}$  is divided into two functions (denoted in this paper as  $G$  and  $H$  functions) having fewer variables, i.e.  $F = H(U, G(V, W))$ . The decomposition scheme is shown in Fig. 1 [3]. The input variables of function  $G$  are called bound variables, while variables from set  $U$  are called free variables. The main problem is to obtain optimal sets of input variables for both functions. Memory utilization results are the best when the number of input variables of function  $H$  equals  $\kappa$ .

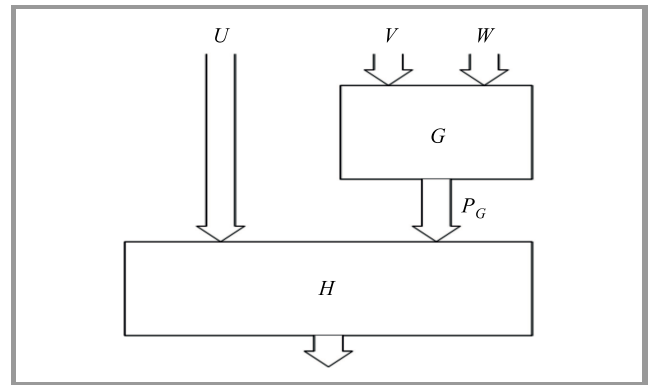


Fig. 1. Functional decomposition scheme.

The notion of partition algebra [8] and  $r$ -admissibility [9] may be used to address the problem of finding sets  $U$  and  $V$  form ( $U \cup V = X$ ,  $U \cap V = \emptyset$ ). However, it is often the case that the introduction of an additional set  $W \subseteq U$  allows to further minimize a Boolean function, even though the number of inputs of function  $G$  was increased. Such a decomposition scheme is called non-disjoint decomposition. Therefore, proposing an efficient method for performing such a decomposition is very important. In particular, the

form of set  $W$  and truth tables of functions  $G$  and  $H$  need to be found.

The exact method of finding functional decomposition consisting in using an SMT solver was proposed in [4]. However, it is a complex solution and may be used only for small values of  $K$ . Therefore, proposing a heuristic method is a very important task. The other method uses a graph coloring problem [3]. This paper examines how the usage of different techniques affects the computation time and quality of the received solution. It provides results obtained for random functions and standard benchmark IGFs, i.e. *M-out-of-N* that do not have an optimum linear decomposition [2] and cannot be minimized using variable reduction algorithms.

The rest of the paper is organized as follows. A brief overview of partition description, partition algebra, and  $r$ -admissibility is presented in Section 2. Section 3 introduces graph theory algorithms and describes how they are applied in finding functional decomposition. An evaluation of the proposed approaches is presented in Section 4. Section 5 concludes the paper.

## 2. Partition Algebra

The notion of partition algebra [8] and  $r$ -admissibility [9] is shortly introduced in this part of the paper. Let  $S$  be a finite set. Consider a collection of subsets of  $S$ . If their union is  $S$ , then we call it a cover of  $S$ . A blanket is a cover  $\mathbb{B} = \{B_1, B_2, \dots, B_k\}$  of a non-empty and distinct subset of  $S$ , called blocks, whose union is  $S$ . A set system is a blanket in which the blocks satisfy that  $B_i \subseteq B_j \Rightarrow i = j$ . A partition is a set system, where the blocks are disjoint, i.e.  $B_i \cap B_j = \emptyset$  ( $i \neq j$ ).

Notice that for IGFs, the  $F(X)$  is an isomorphic function between domain  $D^N$  and the set  $T = \{1, 2, \dots, K\}$ . Each block of partition  $P_a$  includes these elements of the set  $T$  which have the same value at the  $x_a$  position.

Let  $P_a$  and  $P_b$  be two partitions on the same set. We define the following relation  $\leq$  for partitions:

$$P_a \leq P_b \Leftrightarrow \forall B_i \in P_a : (\exists B_j \in P_b : B_i \subseteq B_j) . \quad (3)$$

Multiplication of the partitions may be defined, i.e.  $P = P_a P_b$  if  $P$  is the partition with the largest blocks such that  $P \leq P_a$  and  $P \leq P_b$ . This partition may be easily calculated by finding intersections of all blocks of  $P_a$  and  $P_b$ . The following theorem can be formulated using the theory introduced above.

**Theorem 1:** A function  $F$  has a functional decomposition  $F = H(U, G(V))$  if and only if  $\exists P_G \geq P_V : P_U P_G \leq P_F$  [8].

The quotient partition may be used to determine sets  $U$  and  $V$ . This is a partition of  $P_a$  over  $P_b$  whose blocks are those of  $P_a$  and elements are the elements of the product  $P_a P_b$ . We denote this partition  $P_a | P_a P_b$ . Using the quotient partition, we define  $r$ -admissibility of the set  $\{P_1, P_2, \dots, P_k\}$  in relation to the partition  $P_F$  in the following manner:

$$r = k + \lceil \log_2 (\gamma(P_1 P_2 \dots P_k | P_1 P_2 \dots P_k P_F)) \rceil , \quad (4)$$

where  $\gamma(P_1 P_2 \dots P_k | P_1 P_2 \dots P_k P_F)$  is the number of elements in the largest block of the quotient partition.  $P_F$  corresponds to the characteristic partition of function  $F$ , i.e. the partition whose blocks include these elements that map onto the same value of  $F(X)$ . Notice that for any IGF, the characteristic partition equals  $P_F = \{\overline{1}, \overline{2}, \dots, \overline{K}\}$ .

The concept of  $r$ -admissibility may be interpreted as follows: if a set of partitions is  $r$ -admissible, then there exists a decomposition in which function  $G$  has  $r-k$  outputs, function  $H$  has  $r$  inputs and  $W \subseteq U$  is also an input to function  $G$ . In the best case scenario,  $W = \emptyset$ . By choosing a set  $U$  in which  $r$  is as small as possible ( $V = X \setminus U$ ), we may easily determine the number of input variables of function  $H$ . In that case,  $k = |U|$  and function  $G$  has  $r - |U|$  outputs. A necessary condition for  $r$ -admissibility is presented in lemma 1 [8].

**Lemma 1:** Set  $P$  is  $r$ -admissible if and only if all subsets of  $P$  are  $s$ -admissible, where  $s \leq r$ .

This lemma may be used to easily form set  $U$ . To demonstrate the application of  $r$ -admissibility, let us consider example 1.

Table 1  
Example function

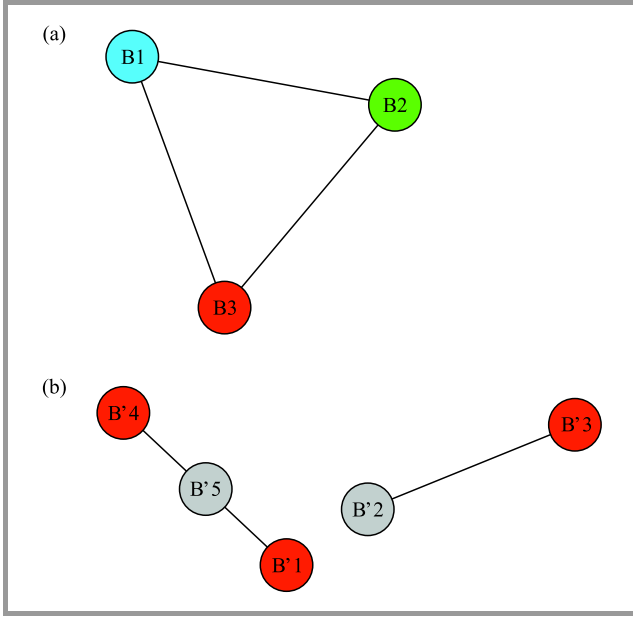
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $F(X)$ |
|-------|-------|-------|-------|--------|
| 0     | 1     | 1     | 0     | 1      |
| 1     | 1     | 0     | 1     | 2      |
| 0     | 0     | 0     | 0     | 3      |
| 1     | 1     | 1     | 0     | 4      |
| 0     | 0     | 0     | 1     | 5      |
| 0     | 1     | 0     | 0     | 6      |

**Example 1:** Consider an IGF shown in Table 1 and notice that  $K = 6$  and  $N = 4$ . The following partitions are calculated:

- $P_1 = \{\overline{1, 3, 5, 6}, \overline{2, 4}\} \Rightarrow r = 1 + \lceil \log_2(4) \rceil = 3,$
- $P_2 = \{\overline{1, 2, 4, 6}, \overline{3, 5}\} \Rightarrow r = 1 + \lceil \log_2(4) \rceil = 3,$
- $P_3 = \{\overline{1, 4}, \overline{2, 3, 5, 6}\} \Rightarrow r = 1 + \lceil \log_2(4) \rceil = 3,$
- $P_4 = \{\overline{2, 5}, \overline{1, 3, 4, 6}\} \Rightarrow r = 1 + \lceil \log_2(4) \rceil = 3.$

All partitions are 3-admissible. Thus, we may use partition multiplication to further search for functional decomposition of an input function. The following partitions are obtained:

- $P_1 P_2 = \{\overline{1, 6}, \overline{3, 5}, \overline{2, 4}\} \Rightarrow r = 2 + \lceil \log_2(2) \rceil = 3,$
- $P_1 P_3 = \{\overline{1}, \overline{3, 5, 6}, \overline{4, 2}\} \Rightarrow r = 2 + \lceil \log_2(3) \rceil = 4,$
- $P_1 P_4 = \{\overline{5}, \overline{1, 3, 6}, \overline{2, 4}\} \Rightarrow r = 2 + \lceil \log_2(3) \rceil = 4,$
- $P_2 P_3 = \{\overline{1, 4}, \overline{2, 6}, \overline{3, 5}\} \Rightarrow r = 2 + \lceil \log_2(2) \rceil = 3,$
- $P_2 P_4 = \{\overline{2, 5}, \overline{1, 3, 4, 6}\} \Rightarrow r = 2 + \lceil \log_2(3) \rceil = 4,$
- $P_3 P_4 = \{\overline{1, 4}, \overline{2, 5}, \overline{3, 6}\} \Rightarrow r = 2 + \lceil \log_2(2) \rceil = 3.$



**Fig. 2.** Graphs obtained in two iterations: (a) graph representing an example function, (b) regenerated graph.

Based on lemma 1, it is known that there is no partition with  $k = 3$  that is 3-admissible. Therefore, the following input variables may be used to form set  $U$ :  $x_1$  and  $x_2$ . Since  $V = X \setminus U$ , we get the following set  $V$ :  $\{x_3, x_4\}$ . Moreover, the number of outputs from function  $G$  equals  $r - |U| = 3 - 2 = 1$ . Two different forms of set  $U$  could be used as well:  $\{x_2, x_3\}$  and  $\{x_3, x_4\}$ . Both these pairs are also 3-admissible. A question regarding the existence of a disjoint decomposition of this function, i.e. whether  $W = \emptyset$ , is discussed in next section.

### 3. Graph Theory and Its Application

As mentioned in the previous section,  $r$ -admissibility may be used to determine the input variables to form  $U$  and  $V$  sets. Additionally, the minimal number of input variables to function  $H$  is found. The graph theory may be used to address two more problems:

1. which variables should form a set  $W$ ?
2. what values are mapped to function  $G$  outputs?

Let  $\Gamma = (N_\Gamma, E_\Gamma)$  be an undirected graph, where each node (called also a vertex)  $v \in N_\Gamma$  represents a block of  $P_V$  and each edge  $e = (B_i, B_j) \in E_\Gamma$  represents an unmergeable pair of  $P_U|P_U P_F$  partition. Two blocks  $B_i$  and  $B_j$  are unmergeable if partition  $P_{ij}$  obtained from  $P_V$  by merging those two blocks satisfies the following condition:  $P_U P_{ij} \not\leq P_F$ . In that case, calculating  $P_G$  (the characteristic partition of function  $G$ ) consists in finding the coloring of the graph. The problem of graph coloring consists in assigning a color to each node, so that no two adjacent nodes have the same color assigned. The same value of function  $G$  is assigned to the nodes colored using the same color.

In order to find non-disjoint functional decomposition by using the graph coloring problem [3], the following steps are required:

1. calculate  $r$ -admissibility of an input function,
2. choose set  $U$  such that  $r$  is at its minimum,
3.  $V = X \setminus U$ ,
4. generate graph  $\Gamma$  based on  $P_U|P_U P_F$  and  $P_V$  partitions,
5. while the chromatic number of the graph  $\chi(\Gamma)$  is larger than  $2^{r-|U|}$  add a variable from set  $U$  to set  $W$ ,
6. unless  $W = U$ , regenerate the graph and go back to the previous step.

It should be remembered that  $r - |U|$  is the number of outputs of function  $G$ . Thus, the number of different values specified with so many variables is  $\beta = 2^{r-|U|}$  at the most. Therefore, the chromatic number of graph  $\Gamma$  cannot be larger than  $\beta$  to represent a function using functional decomposition scheme.

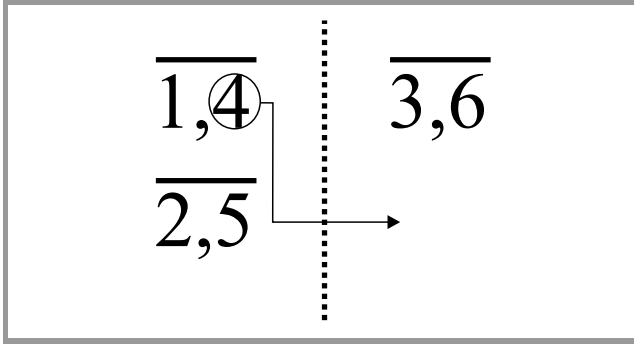
The key part of the presented procedure is choosing the right variable to be added to set  $W$ . In the original paper [3], a node with the maximum degree is chosen. If there are several nodes with the same degree, we choose the first one found. Furthermore, this node must correspond to the block of  $P_V$  partition, which contains at least two elements  $p, q$  that  $\exists e_1, e_2 \in E_\Gamma : \{p\} \in e_1 \wedge \{q\} \in e_2$  and belongs to different blocks in  $P_V P_i$  partition. As a result,  $x_i \in U$  is added to set  $W$ . Different approaches to the choice of the node are analyzed further on in this section.

We use partition multiplication to calculate  $P_{V'}$  partition and regenerate the graph, i.e.  $P_{V'} = P_V P_i$ . This new partition contains more blocks than a partition  $P_V$ . Thus, the regenerated graph has more nodes. On the other hand, the number of edges remains unchanged.

**Example 2:** Consider the function shown in Table 1. As reported in the previous section, we have  $U = \{x_1, x_2\}$ ,  $V = \{x_3, x_4\}$ , and  $r = 3$  through the application of  $r$ -admissibility. By using partition algebra, the following partitions are calculated:

- $P_U = P_1 P_2 = \{\overline{1,6}; \overline{3,5}; \overline{2,4}\},$
- $P_U|P_U P_F = \{\overline{(1)(6)}; \overline{(3)(5)}; \overline{(2)(4)}\},$
- $P_V = \{\overline{1,4}; \overline{2,5}; \overline{3,6}\} = \{B_1, B_2, B_3\}.$

The process of creating  $P_G$  is presented in Fig. 3. It starts with moving blocks  $B_1$  and  $B_2$  to the first block of  $P_G$  and a block  $B_3$  to the second block. However, based on the  $P_U|P_U P_F$  partition, element 4 should be separated from 2. Thus, the minterm  $\overline{1,4}$  needs to be divided. Since vectors 1 and 4 differ at position  $x_1$ , this variable is added to set  $W$ . To automate the process, we construct graph  $\Gamma$ . Notice that we get three blocks  $B_i$  (based on partition  $P_V$ ), which will be represented by nodes in the graph. Furthermore, we get three unmergeable pairs:  $(1,6)$ ,  $(3,5)$  and  $(2,4)$ . Therefore,  $B_1$  is unmergeable with  $B_3$ , since  $1 \in B_1$  and  $6 \in B_3$ .

Fig. 3. Creation of  $P_G$  partition.

Similarly,  $B_3$  is unmergeable with  $B_2$  and  $B_2$  is unmergeable with  $B_1$ . Thus, we get graph  $\Gamma$  presented in Fig. 2a. This graph is 3-colorable. However,  $\chi(\Gamma) = 3 > 2^{r-|U|} = 2$ . Thus, disjoint decomposition of this function does not exist, i.e.  $W \neq \emptyset$ .

In order to find non-disjoint decomposition, we need to add a variable to set  $W$ . We look for a variable that separates  $1 \in B_1$  from  $4 \in B_1$ . Therefore, the node that represents block  $B_1$  will be split into two nodes, which might lead to decreasing the chromatic number of the graph. It may be noticed in Table 1 that vectors one and two differ at position  $x_1$ . Therefore, we add this variable to set  $W$ . Having  $W = \{x_1\}$ , a new partition  $P_{V'}$  is calculated:

$$P_{V'} = P_V P_1 = \{\overline{1;4}; \overline{2;5}; \overline{3;6}\} = \{B'_1, B'_2, B'_3, B'_4, B'_5\}.$$

Partition  $P_U$  remains the same. Graph  $\Gamma$  is now 2-colorable (see Fig. 2b). Thus, non-disjoint decomposition is found. The same output value from function  $G$  is assigned to the nodes colored using the same color. The truth tables obtained are presented in Table 2.

Table 2  
Decomposed function

| Function $G$ |       |       |     | Function $F$ |       |     |     |
|--------------|-------|-------|-----|--------------|-------|-----|-----|
| $x_1$        | $x_3$ | $x_4$ | $G$ | $x_1$        | $x_2$ | $G$ | $F$ |
| 0            | 1     | 0     | 0   | 0            | 1     | 0   | 1   |
| 1            | 0     | 1     | 0   | 1            | 1     | 0   | 2   |
| 0            | 0     | 0     | 1   | 0            | 0     | 1   | 3   |
| 1            | 1     | 0     | 1   | 1            | 1     | 1   | 4   |
| 0            | 0     | 1     | 0   | 0            | 0     | 0   | 5   |
| 0            | 0     | 0     | 1   | 0            | 1     | 1   | 6   |

In the presented procedure, the node with the maximum degree is used to determine the variable to be added to set  $W$ . This method was based on the observation that  $\chi(\Gamma) \leq \Delta(\Gamma) + 1$ , where  $\Delta(\Gamma)$  is the maximum node degree. Thus, reduction of the upper bound on the chromatic number by splitting the node with the maximum degree may lead to a lower value of  $\chi(\Gamma)$ . In this paper, we examine a different approach, i.e. the application of the maximum clique problem. This proposal is based on the fact that  $\chi(\Gamma) \geq \omega(\Gamma)$ , where  $\omega(\Gamma)$  is the size of the maximum

clique. Thus, by choosing a node from that clique, we may decrease the lower bound on the chromatic number.

Clique  $C$  of  $\Gamma$  is a subset of  $N_\Gamma$  such that every two vertices of  $C$  are adjacent, i.e.  $\forall u, v \in C \{u, v\} \in E_\Gamma$ . The maximum clique problem (MCP) consists in finding a clique that is not contained in any other clique and its cardinality is the largest among all cliques. For example, in the graph presented in Fig. 2a, we have clique  $C = \{B_1, B_2, B_3\}$  whose cardinality is equal to 3. This clique is the maximum. On the other hand,  $C' = \{B_1, B_2\}$  is a clique as well, but  $C' \subset C$ . MCP is a well-studied problem. Various approaches have been proposed in the literature [10], both exact and heuristic, to solve this problem. The SageMaths [11] `clique_maximum()` function can be used to find the maximum clique in an undirected graph. It uses the Cliquer software [12] to solve the MCP problem by default, which implements an exact branch-and-bound algorithm. The main weakness of this approach is that no polynomial-time (exact) algorithm is known for the MCP, i.e. it is classified as NP-complete. The fastest algorithm solves the problem in time  $O(2^{0.249 \cdot |N_\Gamma|})$  [13]. Thus, it was decided that a random variable addition to set  $W$  should be analyzed. This method was chosen because it is a less time consuming approach. However, the solution quality might be low. Furthermore, each application of this method may lead to different results.

In paper [3], the exact coloring algorithm using mixed integer linear programming (MILP) was relied upon to find the coloring. However, usage of the exact coloring algorithm does not guarantee that the optimum result will be found [4]. Moreover, its complexity is high, i.e. it is an NP-complete problem. Thus, we examine the application of the heuristic coloring algorithm and its influence on the received results. In particular, the Welsh-Powell [14] algorithm is considered in this paper.

One of the recent studies [15] shows that it is a very efficient algorithm, but the solution quality is sometimes bad. Time complexity of this algorithm is  $O(|N_\Gamma|^2)$ . We will learn how relevant this is in the next section. Any other heuristic or meta-heuristic methods may be used to provide better time-efficiency than the exact coloring algorithm.

## 4. Evaluation

In this section, we evaluate six different approaches:

1. A1 – application of the maximum clique problem and the exact coloring algorithm,

Table 3  
Linear decomposition results

| $K$ | $\kappa$ | $P_{avg}$ | $P_{opt}$ |
|-----|----------|-----------|-----------|
| 20  | 5        | 5.21      | 787       |
| 30  | 5        | 6.16      | 0         |
| 40  | 6        | 6.98      | 20        |
| 50  | 6        | 7.48      | 0         |

Table 4  
Results for different values of  $K$

| Results for $K = 20$ |      |      |      |      |      |      |
|----------------------|------|------|------|------|------|------|
|                      | A1   | A2   | A3   | A4   | A5   | A6   |
| $ W  = 0$            | 16   | 15   | 16   | 15   | 16   | 15   |
| $ W  = 1$            | 103  | 78   | 103  | 67   | 99   | 83   |
| $ W  = 2$            | 80   | 96   | 80   | 102  | 75   | 77   |
| $ W  = 3$            | 14   | 23   | 14   | 29   | 23   | 38   |
| $ W  = 4$            | 0    | 0    | 0    | 0    | 0    | 0    |
| $ W _{avg}$          | 1.43 | 1.60 | 1.43 | 1.68 | 1.49 | 1.65 |
| $ W _{rel}$          | 1.00 | 1.12 | 1.00 | 1.17 | 1.04 | 1.15 |
| $T_{rel}$            | 2.49 | 1.59 | 2.39 | 1.67 | 1.64 | 1.00 |

| Results for $K = 30$ |      |      |      |      |      |      |
|----------------------|------|------|------|------|------|------|
|                      | A1   | A2   | A3   | A4   | A5   | A6   |
| $ W  = 0$            | 590  | 406  | 590  | 406  | 590  | 406  |
| $ W  = 1$            | 259  | 379  | 247  | 373  | 261  | 387  |
| $ W  = 2$            | 133  | 196  | 145  | 201  | 131  | 189  |
| $ W  = 3$            | 3    | 3    | 3    | 3    | 3    | 3    |
| $ W  = 4$            | 0    | 0    | 0    | 0    | 0    | 0    |
| $ W _{avg}$          | 0.54 | 0.79 | 0.55 | 0.80 | 0.54 | 0.79 |
| $ W _{rel}$          | 1.00 | 1.47 | 1.03 | 1.48 | 1.00 | 1.45 |
| $T_{rel}$            | 2.96 | 1.45 | 2.77 | 1.19 | 2.78 | 1.00 |

| Results for $K = 40$ |      |      |      |      |      |      |
|----------------------|------|------|------|------|------|------|
|                      | A1   | A2   | A3   | A4   | A5   | A6   |
| $ W  = 0$            | 81   | 52   | 81   | 51   | 81   | 51   |
| $ W  = 1$            | 210  | 151  | 216  | 165  | 216  | 156  |
| $ W  = 2$            | 326  | 338  | 297  | 305  | 278  | 325  |
| $ W  = 3$            | 294  | 319  | 302  | 337  | 313  | 301  |
| $ W  = 4$            | 69   | 109  | 81   | 124  | 92   | 147  |
| $ W _{avg}$          | 2.06 | 2.29 | 2.09 | 2.34 | 2.12 | 2.34 |
| $ W _{rel}$          | 1.00 | 1.11 | 1.01 | 1.13 | 1.03 | 1.14 |
| $T_{rel}$            | 2.37 | 1.25 | 2.06 | 1.15 | 1.94 | 1.00 |

| Results for $K = 50$ |       |      |       |      |       |      |
|----------------------|-------|------|-------|------|-------|------|
|                      | A1    | A2   | A3    | A4   | A5    | A6   |
| $ W  = 0$            | 540   | 388  | 540   | 388  | 540   | 388  |
| $ W  = 1$            | 270   | 201  | 275   | 208  | 263   | 207  |
| $ W  = 2$            | 141   | 260  | 124   | 246  | 149   | 275  |
| $ W  = 3$            | 40    | 138  | 48    | 142  | 42    | 125  |
| $ W  = 4$            | 4     | 4    | 1     | 1    | 2     | 1    |
| $ W _{avg}$          | 0.69  | 1.16 | 0.68  | 1.15 | 0.70  | 1.14 |
| $ W _{rel}$          | 1.02  | 1.71 | 1.00  | 1.69 | 1.03  | 1.68 |
| $T_{rel}$            | 21.82 | 1.53 | 22.10 | 1.25 | 22.87 | 1.00 |

- A2 – application of the maximum clique problem and a heuristic coloring algorithm,
- A3 – application of finding a node with maximum order and the exact coloring algorithm,
- A4 – application of finding a node with maximum order and a heuristic coloring algorithm.
- A5 – application of random variable selection and the exact coloring algorithm,
- A6 – application of random variable selection and the heuristic coloring algorithm.

In this experiment we generated random IGFs with  $N = 32$  and  $K = \{20, 30, 40, 50\}$ . For each value of  $K$ , 1000 functions were generated. All functions were firstly minimized using the linear decomposition algorithm [2]. The results obtained were presented in Table 3.  $P_{avg}$  denotes the average number of variables after linear decomposition and  $P_{opt}$  denotes the number of functions that have an optimum linear decomposition. For those functions that do not have an optimum linear decomposition, we looked for functional decomposition. Table 4 summarizes the results obtained. The following parameters are presented:

- $|W| = 0, 1, \dots, 4$  – the number of functions with a particular size of set  $W$ ,
- $|W|_{avg}$  – the average size of set  $W$ ,
- $|W|_{rel}$  – the value calculated using the following equation:  $W_{avg}/W_{avg}^*$ , where  $W_{avg}^*$  is the minimum average size among all approaches,
- $T_{rel}$  – the average computation time for a particular approach divided by the minimum average computation time of all approaches.

The most remarkable result to emerge from the data is that the application of MILP has led to the lowest average size of set  $W$ . No significant difference in the obtained results was observed between A1, A3, and A5 approaches. However, the approach proposed in the original paper [3] provided the best results only for  $K = 20$  and  $K = 50$ . On average, the application of MCP has led to the best results, i.e. value of  $|W|_{rel}$  obtained using this approach is the lowest for three different values of  $K$ .

Even though solution quality is very good, all these approaches are very time consuming. The average computation time for different values of  $K$  is presented in Fig. 4. The results presented confirm usefulness of the Welsh-Powell algorithm. Approaches relying on this algorithm



guarantee the lowest computation time. Moreover, the computation time does not increase significantly depending on the value of  $K$ . This study shows that selection of the graph coloring algorithm is much more important than the approach adopted to select the variables. The use of a heuristic graph coloring algorithm leads to slightly worse solution quality. However, time complexity is much better, e.g. approaches relying on the Welsh-Powell algorithm were over 20 times faster for  $K = 50$ .

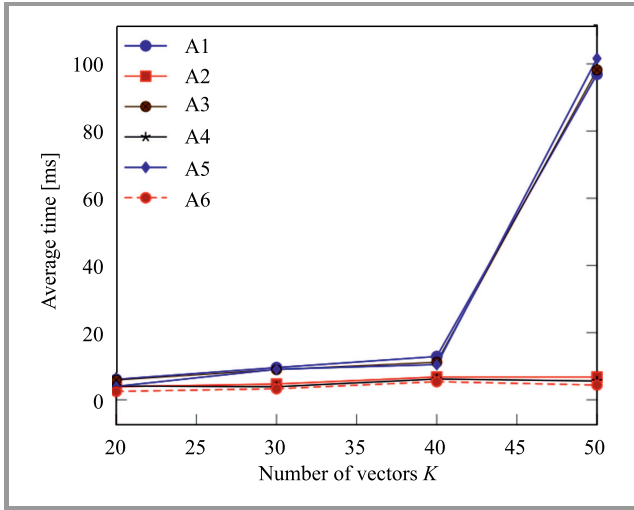


Fig. 4. Average computation time for different values of  $K$ .

Surprisingly, the number of functions whose decomposition led to  $|W| = 4$  is large for  $K = 40$ . However, that number is close to zero for other values of  $K$ . The number of functions with  $|W| = 2$  and  $|W| = 3$  is also quite high. Thus, the average size of set  $W$  is significantly higher. The reason for this rather extraordinary result is still unclear.

Table 5  
Number of minimized functions using functional decomposition

| $K$ | A1  | A2  | A3  | A4  | A5  | A6  |
|-----|-----|-----|-----|-----|-----|-----|
| 20  | 213 | 212 | 213 | 213 | 213 | 213 |
| 30  | 985 | 984 | 985 | 983 | 985 | 985 |
| 40  | 980 | 969 | 977 | 973 | 980 | 980 |
| 50  | 995 | 991 | 988 | 985 | 996 | 996 |

The number of minimized functions using each approach is presented in Table 5. For  $K = 20$ , most functions have the optimum linear decomposition. Thus, we did not search for the functional decomposition of those functions. Notice that the second approach has led to one less successful functional decomposition, i.e. decomposition was not found because  $W = U$ . For higher values of  $K$ , most of the functions were minimized using the approaches described in this paper. Notice that the use of the exact coloring algorithm typically leads to a higher number of minimized functions. The single most significant observation to emerge from the

results is that approaches A5 and A6 approaches result in the highest number.

Interestingly, for  $K = 20$  and  $K = 40$ , we were able to find an optimum linear (Table 3) or a functional decomposition of every input function using approaches A1, A5, or A6 approaches.

The experiments confirm the assumption that random variable selection is time-efficient. The fastest approach for all values of  $K$  was A6. Contrary to expectations, quality of the solution obtained while using this approach was relatively good. However, this approach leads to different results in each execution of the algorithm.

Consider  $M$ -out-of- $N$  functions. They are often chosen as benchmarks functions to evaluate the efficiency of IGF decomposition algorithms. They consist of  $K = \binom{N}{M}$  vectors, whose length is  $N$  and Hamming weight is  $M$ . It was proved [1] that the optimum linear decomposition of a 2-out-of-20 coder does not exist. To the best of our knowledge, the optimum decomposition of a 4-out-of-20 function was not found either [2]. We use the described approaches to further minimize both functions. The inputs to the algorithm are truth tables of both functions after linear decomposition. The results obtained were presented in Table 6, where “–” means that the algorithm did not stop within the allocated time, i.e. the period of 60 minutes. Notice that the use of MILP for  $M = 4$  has led to timeouts. Thus, the presented results prove that IGFs with many vectors should be minimized using a heuristic graph coloring algorithm.

In this paper, we do not analyze other standard benchmark functions, i.e.  $M$ -out-of-16, 1-out-of-20, and 3-out-of-20 functions. Those functions have optimum linear [1], [2] or disjoint functional decomposition [4].

Table 6  
Results for  $M$ -out-of-20 functions

| Approach | $M = 2, K = 190$ |       | $M = 4, K = 4845$ |       |
|----------|------------------|-------|-------------------|-------|
|          | Time [s]         | $ W $ | Time [s]          | $ W $ |
| A1       | 0.17             | 1     | –                 | –     |
| A2       | 0.05             | 2     | 19.33             | 3     |
| A3       | 0.25             | 2     | –                 | –     |
| A4       | 0.04             | 2     | 17.92             | 3     |
| A5       | 0.17             | 1–3   | –                 | –     |
| A6       | 0.02             | 2–3   | 7.98              | 2–4   |

For each approach, the computation time and the result obtained were presented. We analyze only the computation time of steps 3–5 from the procedure described in Section 3. We run the algorithm ten times, applying approaches A5 and A6. In the table, we present the minimum and the maximum number of variables in set  $W$ . Moreover, we present the time needed to find the solution with the minimum size of set  $W$ . Notice that for  $M = 4$ , the size of  $W$  might be equal to 2, 3 or 4. Similarly, for  $M = 2$ , quality of the solution varies. Thus, the algorithm relying

on random variable selection needs to be executed multiple times in order to find the best solution.

The table is interesting in several ways. As far as we know, no other authors have found non-disjoint functional decomposition with  $|W| = 1$  for a 2-out-of-20 function using a heuristic approach. Moreover, this solution is optimal. It was proved using the SMT-based exact method [4]. We also believe that the result obtained for a 4-out-of-20 function, i.e.  $|W| = 2$ , is the one that is best-known.

In order to highlight the significance of the results obtained, we analyze the total memory size using the scheme presented in Fig. 1. If both functions  $G$  and  $H$  are implemented using memories, the total size in bits equals:

$$\mathbb{S} = 2^{|V \cup W|} \cdot (r - |U|) + 2^r \cdot \kappa. \quad (5)$$

The results obtained are presented in Table 7. On average, the memory size was reduced by 10% using the approach presented in this paper due to the minimization of set  $W$  size. The results presented highlight the usefulness of the approach proposed in this paper.

Table 7  
Memory size ( $\mathbb{S}$ ) for  $M$ -out-of-20 functions

| $M$ | Paper      | $ U $ | $ V $ | $ W $ | $r$ | $\mathbb{S}$ |
|-----|------------|-------|-------|-------|-----|--------------|
| 2   | [4]        | 4     | 5     | 2     | 8   | 2560         |
|     | This paper | 4     | 5     | 1     | 8   | 2304         |
| 4   | [4]        | 6     | 9     | 3     | 13  | 135168       |
|     | This paper | 6     | 9     | 2     | 13  | 120832       |

On the other hand, the memory size using linear decomposition only, equals:

$$\mathbb{S}_L = 2^p \cdot \kappa, \quad (6)$$

where  $p$  denotes the number of variables after linear decomposition, i.e.  $K = 20 \Rightarrow p = 9$  and  $K = 40 \Rightarrow p = 15$  [2]. The memory size for  $K = 20$  is  $\mathbb{S}_L = 4096$  bits. Functional decomposition has led to minimizing memory usage by 43.75%. Moreover, for  $K = 40$ , minimization by 71.63% has been achieved, i.e.  $\mathbb{S}_L = 425984$ . These results prove how important functional decomposition is.

During the experiment, we also tested whether the first-fit approach to variable selection leads to good results, i.e. variables were added in order of occurrence in set  $U$ . Our results were below expectations in terms of solution quality. For example, we obtained  $|W| = 4$  for both 2-out-of-20 and 4-out-of-20 functions using the Welsh-Powell algorithm. On the other hand, this method was on average faster than other approaches. Since the solution quality achieved is bad, we decided to omit the results.

## 5. Conclusion

In this paper, application of the graph theory in non-disjoint decomposition of index generation functions was investigated. Several possible modifications of the original method were described [3] and the results obtained were analyzed.

The research has highlighted the importance of selection of the graph coloring algorithm selection. In general, the results prove that methods using the Welsh-Powell algorithm are time-efficient and provide relatively good solution quality. The method using random variable selection to set  $W$  was the fastest one. On the other hand, approaches using the exact coloring algorithm led to the best results in terms of solution quality. However, they are much more time-consuming and led to timeouts for functions with a large set of input vectors.

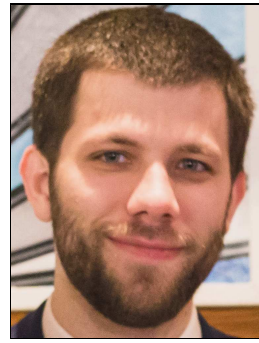
Novel solutions were found for non-disjoint decomposition of standard benchmark functions, i.e. 2-out-of-20 and 4-out-of-20 functions. They guarantee minimization of memory usage by approximately 10% compared to previous results [4]. These results have further strengthened confidence in the usefulness of functional decomposition. A significant minimization of memory requirement is achieved compared to a scenario in which linear decomposition is applied only.

The approach relying on random variable selection is very fast and may lead to very good solutions. This method has some limitations stemming from the fact that each execution of the algorithm might lead to a different result. Therefore, this approach needs to be applied multiple times in order to assess the quality of the provided solution.

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# Review of Experimental Verification Methods of Gyrotron Quasi-optical Mode Converters

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<https://doi.org/10.26636/jiit.2020.141320>

**Abstract**—This survey presents a review of experimental methods relied upon while implementing gyrotron higher mode generation techniques and performing near electromagnetic field measurements in launcher and quasi-optical mode converters. In particular, the paper focuses on low power (cold) testing of gyrotron quasi-optical mode converters outside of the gyrotron, without the presence of high electromagnetic power and electron beams.

**Keywords**—gyrotron, high order modes, measurement for quasi-optical systems.

## 1. Introduction

A gyrotron is a high-power microwave source of coherent electromagnetic radiation, with its frequency within the gigahertz to terahertz range, and with the pulse power level of 50 to 500 kW. The device has a broad range of potential applications, as it may be used, for instance, in nuclear magnetic resonance, plasma diagnostics, spectroscopy, weather monitoring, material processing, high-resolution radar, etc. The operating principle of a gyrotron is based on the electron cyclotron resonance phenomenon [1]–[3]. Gyrotrons use the interaction between an annular electron beam and azimuthal electric field of a circular waveguide mode. Electrons that have a cyclotron frequency set slightly below the resonant frequency of the microwave excited in the cavity, electromagnetic wave takes place. The high-order

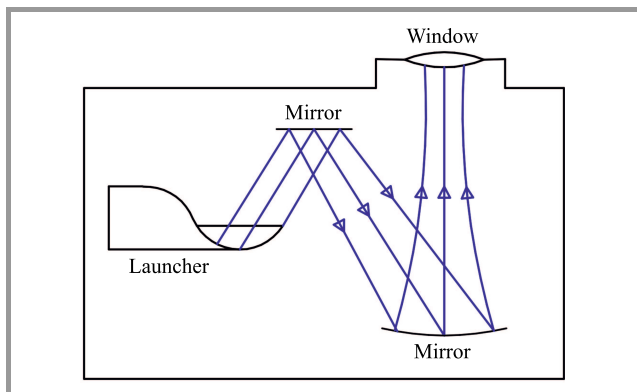
mode EM wave generated within the resonant cavity is converted to a wave beam by the mode converter, is then shaped by mirrors and sent towards the output through a window (Fig. 1).

In order to design quasi-optical mode converters (QoMC), dimensions and the geometry of the mode converter, launcher and mirror system need to be determined. The commonly used analytical equations or graphic methods, such as geometrical optics, allow to obtain preliminary results with a good level of accuracy [4], [5]. These approximate usually serve as a good starting point for further full-wave EM field detailed analysis, performed based on computer simulations. A vector diffraction theory is often applied to simulate converter operation and to predict output beam characteristics. The final stage of the design process consists in experimental verification of the theoretical models. Those examples that are most representative for this specific area of research have been selected and are described in this paper.

## 2. General Remarks on Measurement Setup

Converter design-related mistakes and implementation phase errors need to be eliminated in order to avoid any further problems. Any errors in the QoMC design should be detected before the device starts to operate at maximum power output. Due to high manufacturing costs and considerable health risks associated with powerful gyrotron emissions and the vacuum environment, the so-called cold test method is the only practical method, as it may be deployed without the presence of high levels of EM power [6]–[8]. This method involves measurements of important parameters at safe, low power EM levels. The measurements allow to detect any mistakes made at the design stage or any faults appearing as a result of converter component manufacturing or assembly processes.

A number of methods have been developed to evaluate the performance of the gyrotron QoMC setup. The higher order modes generator is a key element of any “cold test” measuring system. The whispering gallery field mode created by the mode generator is further transformed into



**Fig. 1.** Operating principle of a typical quasi-optical mode converter.

a Gaussian beam using the launcher unit and a mirror system. The standard cold method (from high order mode generator to Gauss beam) requires the design of a higher order mode generator excited by a signal from the vector network analyzer (VNA) instrument, through a set of current sources. In a typical test system, the higher order mode generator is connected between the VNA TX output and the QoMC input tested. The methods of generating rotating and non-rotating modes are further described in Subsection 3.3.

In some types of measurements, the generators may be omitted. It may be assumed that passive quasi-optical systems are reciprocal. This means that lab-class models may be developed based on the method proposed by Vlasov [9]. In this "reversed" design method, the Gaussian beam created in the horn type antenna is further converted into higher order EM modes in the gyrotron resonator, after passing through the mirror system and launcher unit. The next step in such an approach is to design a system that allows to measure these higher order modes. This solution often relies on two identical converter paths connected with each other in both directions (back-to-back) [10]. Such a system is convenient for transmittance measurements. In this setup, fundamental waveguide modes are available at input and output ports, but the transmittance measured in such a setup is two times lower than that of a single converter. The measurements may be easily performed using the VNA instrument. The mode generator and QoMC components are located very close to each other. To measure the fields radiated by them, the near field measurements methods, using VNA, may be applied. The process is described in greater detail in Section 5. In order to gain insight into the process of field transformation occurring in the QoMC structure, radiation characteristics of the launcher device and the set of mirrors must be determined. This allows to see how the system is changing the geometry of the radiated wave beam. The most important QoMC parameters that should be measured are the radiation characteristics of the launcher and mirrors set. For this purpose, well-known techniques developed for the purpose of antenna near fields measurements are used.

### 3. Generators of Higher Order Modes

The higher order mode generator is often described in the literature as a converter. This is because of the operating principle, pursuant to which fundamental mode is converted in the waveguide in to a higher order axial mode. The following basic parameters determining the quality of the converter may be distinguished. The first one is conversion efficiency, understood as the ratio between the power of the desired output wave and the power of the input wave, with a gyrotron type of field. Purity is the second important parameter, as it determines the percentage of components related to unwanted field types, in the output spectrum. Bandwidth and physical length of the conversion system are its remaining characteristics. The mode generator allows to create appropriate, higher order azimuthal EM field

types, precisely corresponding to those appearing in the gyrotron resonator. Two types of generators are used here. The rotating mode generator (RMG) in which a higher order rotating mode is created is the first of those types. This is a complicated device with low conversion efficiency, but high mode purity. The second type has the form of a non-rotating mode generator (NRMG) [11] in which a non-rotating, higher order mode is created first. This mode is further converted into a rotating mode of the same order using the so-called polarization converter (polarizer). Generators of this type are usually used to generate extremely high order modes (such as  $TE_{22,6}$  etc.) [12]. Such devices have a simple design resulting in high immunity to mechanical interference from vibration. Their significant drawback consists in the extremely long waveguide length and, consequently, narrow bandwidth.

Currently, several known methods of generating higher order modes exist:

- converters based on the principle of periodically disturbance of the geometry of the waveguide wall, which gradually transforms the basic type into a higher order waveguide [12]–[14],
- converters inducing the higher order bay mode through the perforated side wall of the bay (translucent cavity wall), using a fader in the form of a quasi-optical mirror [15],
- converters using Y type power dividers to generate relatively low order modes, such as  $TE_{0,1}$ ,  $TE_{2,1}$ ,  $TE_{4,1}$  [16]. Units of type offer high conversion efficiency, high signal purity and high bandwidth. Due to their symmetrical coupling structure, they may only generate fields that have even azimuth indices of low order fields, such as circular type  $TE_{2,1}$  or  $TE_{4,1}$ .

#### 3.1. Excitation of Selected Higher Order Modes

Excitation of the selected mode may be achieved by using properly spaced electric or magnetic current sources. The current sources may be created by excitation probes or loops, as well as by coupling apertures in waveguide wall. The electromagnetic field inside a circular waveguide is excited by all induced magnetic moments  $P_m$ . The method of exciting modes from any electric or magnetic source is described in [17]. In a typical solution presented in [18], [19], rectangular waveguide sections with the fundamental  $TE_{1,0}$  mode are connected to a circular waveguide via coupling apertures on the waveguide's side wall. To excite a pure  $TE_{m,1}$  mode, magnetic current sources must be applied. For example, to excite  $TE_{2,1}$  and  $TE_{4,1}$  modes, two and four coupling apertures have to be used, respectively. For excitation of the mode which has a circular distribution of the electric field, selective use of two, four or even more magnetic dipoles may be required.

#### 3.2. Excitation of Rotating Higher Order Modes

Non-rotating modes may be converted into rotating modes of the same-order using the so-called polarization con-

verter [20]. The reported conversion efficiency in this method exceeds 63%, with  $-2$  dB within the 92 to 95 GHz range [20]. The EM wave with circular polarization may be decomposed into two waves with linear polarization and equal amplitudes, being shifted in phase by  $90^\circ$ , however. It follows that the  $TE_{m,1}$  wave with circular polarization may be generated by the  $TE_{m,1}$  wave with linear polarization and its degenerated wave  $TE_{m,1}$ . The linearly polarized  $TE_{m,1}$  wave propagates along a circular waveguide and reaches the polarizer. As a polarization converter, a mode converter with azimuth wave rumbling is used. The polarizer radius is expressed as follows:

$$r(\varphi) = a_0 + a_1 \cos(10\varphi), \quad (1)$$

where  $a_0$  is the average polarizer radius,  $a_1$  is the deformation and  $a_0 \gg a_1$ . The  $TE_{m,1}$  wave with linear polarization is gradually transformed into its degenerate wave  $TE_{m,1}$  due to deformation of the waveguide wall. The length of the deformation, the amplitude of the deformation and the length of the transition section are optimized to obtain two output modes of equal amplitude, but with phase shifted by  $90^\circ$ . With higher deformation, the polarizer length will decrease, but the difficulty of manufacturing increases significantly. Two types of rotating mode generators may be distinguished: high-order rotating mode generators with a perforated cavity wall [21], [22] and rotating mode generators with linear mode conversion [23].

### 3.3. General Remarks on Mode Generator Design

The complexity of design problems increases along with the number of mode orders to be excited. When a selected mode is excited, other types with a lower cut-off frequency can also be excited. For example, to excite the  $TE_{4,1}$  mode, you must consider the possibility of excitation of as many as 14 undesirable modes. This means that when exciting the  $TE_{4,1}$  mode, the concentration of the remaining 14 modes should be kept as low as possible. There are several dozen different undesirable modes within the terahertz frequency range. For practical use, the most suitable ones are those for which a large separation between neighboring modes can

be achieved, thus leading to high modal purity. Conversion losses caused by dissipation of ohmic energy on the copper walls of the converter may be a serious problem as well. However, for higher order mode generators, transmission efficiency is usually less important than modal purity.

The first step in the converter design procedure is to select the circular waveguide diameter. The cut-off frequencies for subsequent modes increase as the waveguide diameter decreases (Fig. 2). To increase separation between successive modes, a coaxial waveguide with a center conductor may be used. The presence of an internal conductor has an effect on the spectrum of the modes' eigenfrequencies. This makes the coaxial waveguides spectrum much more spread compared to circular waveguides.

The suppression of undesirable modes near the working mode can be achieved by selecting the appropriate diameter ratio  $C$ :

$$C = \frac{r_{out}}{r_{in}}, \quad (2)$$

where  $r_{out}$  and  $r_{in}$  are the radiuses of the outer conductor and inner coaxial waveguide. More specifically, the coaxial waveguide with a properly selected  $C$  ratio should support the propagation of the desired mode and block the propagation of higher order modes.

For a coaxial waveguide with  $C$  ratio propagation characteristics for the selected mode,  $TE_{m,n}$  may be obtained by solving the following equation [21]:

$$J'_m(\chi_{mn})Y'_m\left(\frac{\chi_{mn}}{C}\right) - J'_m\left(\frac{\chi_{mn}}{C}\right)Y'_m(\chi_{mn}) = 0. \quad (3)$$

When the radius of the outer conductor is much larger than the radius of the center conductor, then:

$$\frac{\chi_{mn}}{C} \rightarrow \infty, \quad (4)$$

and the equation for the boundary conditions of the coaxial waveguide cross-section is simplified to:

$$J'_m(\chi_{mn}) = 0, \quad (5)$$

which determines the cut-off frequency for a field type in a circular waveguide. In practice, a smooth tapered transition is often used to gradually increase the  $C$  ratio for external and internal conductors (Fig. 3). The waveguide and mode converter designed in this way create an open resonance structure that can further improve modal purity of the desired field type.

To avoid the competition effect between modes, a resonator with a high  $Q$  factor should be designed. The high value of the resonator additionally allows to reach the operating point with a high coupling efficiency of the beam with an RF field [23]. To handle higher power levels, the size of the transverse cavity must be increased, which simultaneously lowers coupling efficiency of the beam and radiation levels. The cavity quality factor may be expressed as [23]:

$$Q = 4 \cdot \pi \left( \frac{L^2}{\lambda^2} \right) \cdot \left( \frac{1}{1-\rho} \right), \quad (6)$$

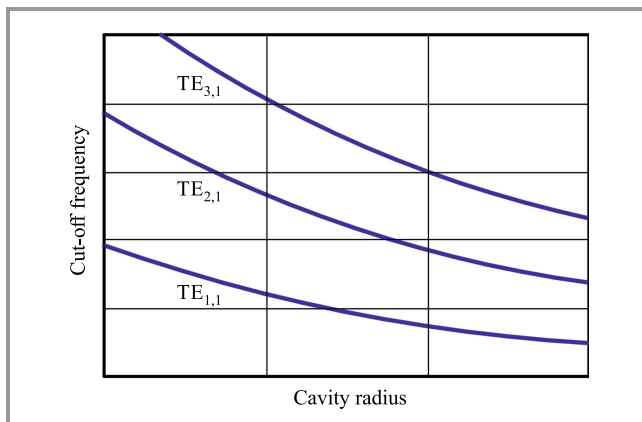
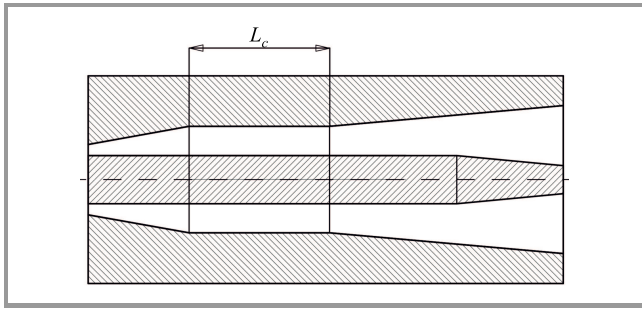


Fig. 2. Circular waveguide cut-off frequency as a function of waveguide radius.





**Fig. 3.** Circular waveguide with tapered transitions and internal conductor.

where  $L$  is the effective length of cavity and  $\rho$  is the reflection coefficient.

Mechanical design is essential for the mode generator. It is important to maintain both simplicity of the structure and the resulting resistance to mechanical interference and vibrations. An important disadvantage of generators with a complicated mechanical design is the need to perform precise adjustments of the quasi-optical elements before commissioning the devices. At higher frequencies within the terahertz range, the elements have to be fabricated with the level of accuracy down to single micrometers. This technology is currently under development [24].

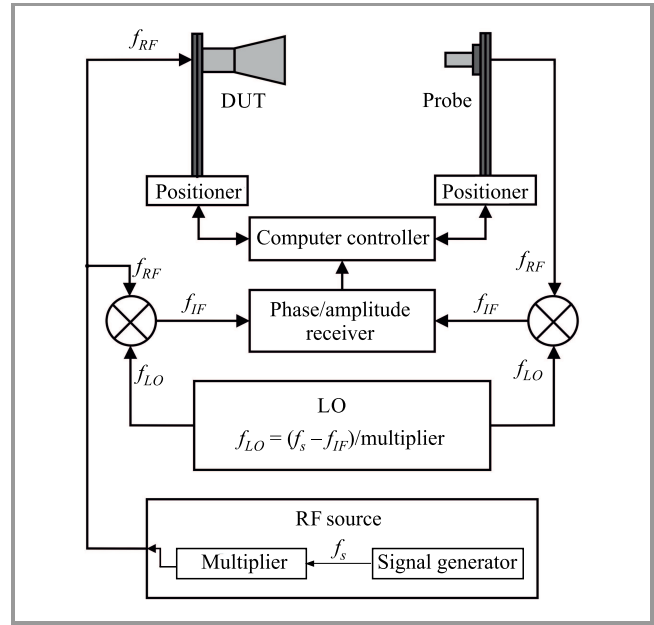
#### 4. Measurements for Quasi-Optical Systems

Individual components of the QoMC converter are usually positioned close to each other. Therefore, for measuring EM fields, near-field measurement methods have been developed. The term “near-field” is defined as the distance from the antenna, remaining in the range of  $\frac{\lambda}{2\pi}$  to  $\frac{2D^2}{\lambda}$  [25]. A typical measuring setup used to measure antennas or devices in the near field is shown in Fig. 4.

The system consists of an RF signal source, two frequency mixers with a properly selected local oscillator and a phase/amplitude receiver. All those components constitute a simplified VNA unit [26]). There is also the tested device (DUT) and the measuring probe. Two computer-controlled electro-mechanical positioners are responsible for the appropriate scanning geometry.

The measurements of field distribution in the entire QoMC are reduced to scanning individual elements, assuming that their location in space is appropriate. This means that starting from the first element, e.g. the Vlasov launcher in the gyrotron’s internal mode converter, the distribution of radiation at the place of attachment of the second element (quasi-elliptical cylinder mirror) is checked first, and then the field reflected from the second element is measured at the location of the third element (mirror phase correction), and so on. Based on such a procedure, the entire quasi-optical system may be characterized, making it much easier to detect possible errors and allowing to quickly introduce some adjustments to the design at the cold testing

stage. It should be noted that adding another component of the QoMC requires that the scanner’s orientation be adjusted relative to the tested system. The direction of the scan axis, which serves as a reference point for starting the scanning procedure, must coincide with the direction of radiation from the tested components, maintaining the radial representation of geometrical optics [27], [28]. Scanning systems comprising different quasi-optic elements (QoE) use different scanning geometries, and implementation of the right geometry may be difficult and complicated.



**Fig. 4.** Block diagram of the measuring system used to measure antennas/devices in the near field.

In classic field scanners, there are three basic scanning geometries – planar, spherical and cylindrical, for which different solutions are provided [25], [29], [30]. A robotic arm may be used for the above cases and for unusual surface mapping with the right number of degrees of freedom [31], [32]. For example, helical (in a cylindrical coordinate system) or spiral (in Cartesian and spherical coordinate systems) scanning techniques significantly reduce the scanning time of antennas in the near field [33]. This is important when we want to quickly perform a near-to-far field transformation, but appropriate non-redundant transformations based on the interpolation method should be used [33]–[41]. In the case of a quasi-optical mode converter, the far field may be taken into account when considering radiation outside the gyrotron window. Regardless of the scanning method used in the measurement of near-field, many factors need to be taken into consideration, including [29]: absorber and its placement, cross-talk and leakage, cable flexing, noise and dynamic range, nonlinearity, normalization, scan area truncation, sampling criteria, separation distance between the test antenna and the probe, probe selection and correction.

Most near-field scanning systems are capable of accurately measuring amplitude, while measuring the phase of radi-

ation patterns at millimeter-wave frequencies still remains a challenging problem [42]. In the millimeter and submillimeter wave ranges, the conventional heterodyne receiver is not immune to phase errors introduced by wiring and by environmental conditions. Generally, amplitude and phase correction [43], [44] should be applied, for example by using multiple receivers [45]–[47] or by applying a phase recovery algorithm from planar scanner maps at different distances from the DUT [48], [49]. This is important in the case of high order rotating gyrotron modes, where phase information is necessary for a good analysis of the quasi-optics mode converter.

#### 4.1. Planar Scanning

As the name implies, a planar scanner (Fig. 5) senses radiation on a plane rather than a rotating surface, which greatly simplifies its design. In addition, most of the beam fields are limited to halfspace [50], and its efficiency in data collection and processing [51] makes it suitable for the work of many research groups [30], [52]–[55]. QoE is most often stationary here, while the probe (scanning antenna) is moved along the  $x$ - $y$  plane. The movement is performed line-by-line, the results of the experiment are taken in the region where the disappearing waves (non-propagative types of field) are not important and the probe position increments less than  $\lambda/2$  in both  $x$  and  $y$  directions [29]. This is necessary in order to meet the Nyquist sampling criterion and to avoid aliasing. Due to the limited measuring surface, planar scanning is suitable for measurements at the output taper of mode generator (before entering the launcher). Each scanning point  $P$  is represented with the use of Cartesian coordinates  $x$ ,  $y$ ,  $z$ , where  $z = d = \text{constans}$ .

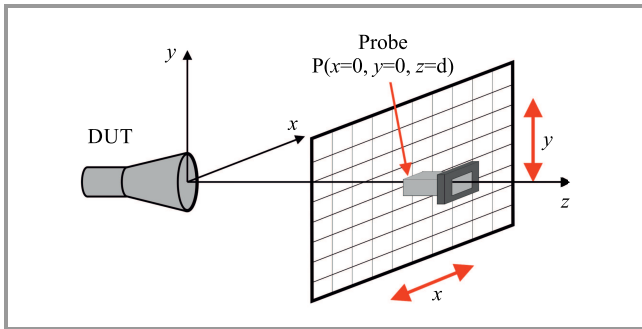


Fig. 5. Near-field planar  $x$ - $y$  scanner.

In publication [56], a four-axis configuration of a planar scanner ( $x$ ,  $y$ ,  $z$ , rotation) is used, with the device also being capable of serving as a spherical scanner, but for a far-field distribution. The planar scanner was used to measure field distribution from the gyrotron output, after attenuation of the signal, with a 2 Hz repetition rate, 140 GHz, mode  $TE_{0,1}$ . The radiation pattern was measured from the “snake” to verify the conversion of the  $TE_{0,1}$  mode to  $TE_{1,1}$ . Many other researchers also use the type of scanning geometry described in [30], where the scanner design allows

to measure quasi-optical elements in the frequency range of 110–220 GHz [30], [57], [58]. In paper [52], researchers used a planar scanner for mirror design for high-power gyrotrons. Field intensity measurements of the third mirror and exit window were also carried out by authors in [44]. Other scientists using EO probes also performed measurements of field distribution in the near-field, at an extremely small distance from the converter aperture, on its outer and inner sides, and measurements of the total mode transformation inside the mode converter. The position of the EO probe is controlled by a programmable table, enabling it to be slid along the  $x$ - $y$ - $z$  axes [45], [46]. Planar geometry is also used for scanning the beam at the output of the pre-bunching converter [28]. Planar scanners are also used to measure field distribution of quasi-optical generators [22].

#### 4.2. Cylindrical Scanning

Usually, the scanner uses a positioner that rotates QoE along the angular coordinate  $\varphi$  and has a linear scanner (motor) moving the probe along the  $y$  axis (Fig. 6).

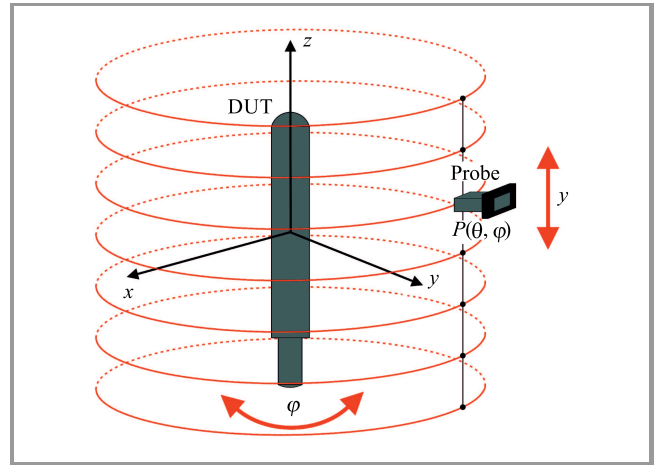


Fig. 6. Cylindrical scanner concept – DUT/QoE rotator for azimuth rotation and probe of the linear  $y$  scanner.

The combination of QoE azimuth rotation and a linear probe is equivalent to scanning on a cylindrical surface around QoE. In practice, two measurement scenarios are considered. The first is requires smooth QoE rotation and change of the probe position. The other is requires a smooth probe sweep along the  $y$  axis and angular QoE rotation. The cylindrical scanning method is particularly useful for testing fan-beam antennas that have a wide beam in one plane, but a narrow beam in the orthogonal plane [29]. Detailed considerations for this type of scan may be found in papers [29], [33], [59]. The author of [27], [60], measured the filed radiation pattern from the Vlasov launcher at the first mirror location. The study was performed at power level in the range of megawatts, 3  $\mu$ s pulsed gyrotron operating in the  $TE_{22,6}$  mode at 146 GHz. A very similar scan was performed by the scientist in [61]. The measurements were made for a megawatt gyrotron operating at 148.8 GHz in the  $TE_{16,2}$  mode.

In all methods, EM field is scanned utilizing a two- or three-axial linear movement unit. Either the probe or QoE are installed on the linear actuator, allowing pixel-by-pixel scanning. In some cases, there is a need to perform image post-processing in order to take into account probe characteristics, scanning curvature or other influences.

#### 4.3. Spherical Scanning

As far as antenna technologies are concerned, the spherical scanning method is widely described in [29], [33], [62], [63] and is used quite often, too. The working principle of the scanner is shown in Fig. 7.

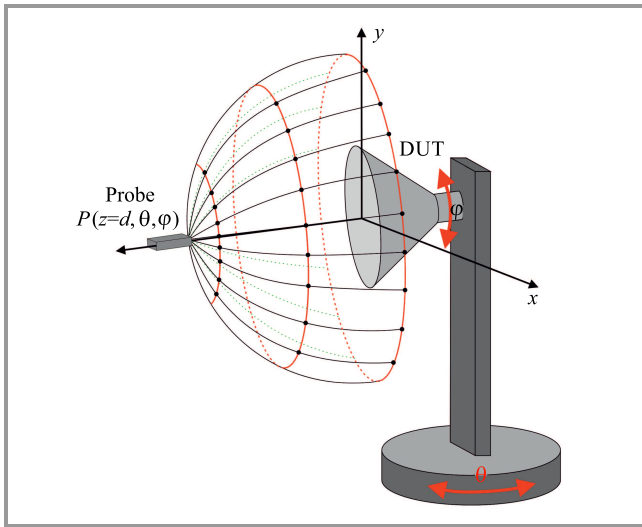


Fig. 7. Spherical scanner: principle of operation.

In the literature concerning QoMC, no examples were found to indicate its use for near-field measurements. Therefore, this type of scanning is performed at the output of the QoMC, at a limited angle of  $6^\circ$  and at a distance exceeding near-field limitations [50]. An example is the snake QoMC carried out in the range of  $\theta \pm 30^\circ$  (sphere segment). The scanner was deployed by placing the Labjack on a pivot arm, where the pivot point was below the waveguide output aperture. Spherical scanning in the far-field has also been used and researched in [62] to determine the optimal cutting angle for the Vlasov mod converter. However, we point to the possibility of using spherical geometry, e.g. for scanning the hyperboloid or paraboloid surface that corresponds to the mirror surface. In addition, by using transformation, the spherical surface may be transformed into other analytically defined surfaces.

## 5. Measurement Probes

The probes are the most important part in the electromagnetic near-field measurement setup. Selection of the appropriate probe type and its dimensions determines which component of the EM field may be measured and with what spatial resolution [64].

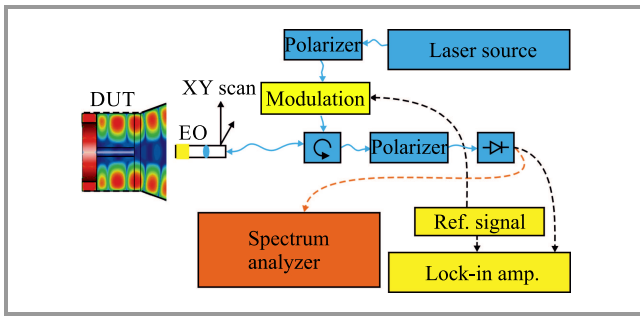
The most common types of probes include miniature horn antennas, open-ended waveguides [65] and coaxial probes with one or two (differential probes) coaxial cables [64]. Open-ended probes are used to measure the electrical component, while closed loop probes are used for the magnetic component [66]. Solutions are also in place using electro-optical effects, allowing for very accurate measurement of EM field distributions.

For example, in [64], the use of an optical probe allowed the authors to reduce spatial resolution 5 times compared to monopole coaxial probes (from 500 to 100  $\mu\text{m}$ ). These types of probes are especially suitable for performing measuring in microelectronics [67]–[69] and for measuring the phase of QoE. No other types of probes provide such good results that would be as close to the results of numerical simulations relying on full-wave description [70].

The use of EO probes to identify whispering gyron gallery mode patterns [55] is the best example here. This paper compares  $\text{TE}_{6,2}$  mode generator measurements obtained using the VNA scan system employing the WR08 open-ended waveguide probe, using EO probes and micro-EO probes, with the results of CST Microwave Studio simulation data. The results show significant limitations of near-field measurements performed with the use of open-ended probe systems. Despite good SNR levels, resolution and accuracy of the measurements are worse and depend on the size of the probe itself and on its large metal surface exposed. Micro-EO probes provided accurate magnitude and phase measurements identifying mode rotation data in the near field. The authors indicate that mode purity values of CW and CCW rotation equaled 98.11% and 99.8%, respectively. In addition, electro-optical probes, as the only ones listed here, allow to measure the field inside such structures as mode generators, converters, and resonators [13]. In the case of QoE measurements performed in the gyrotron, the TE field type is most often used. However, it should be borne in mind that designs exist supporting the TM field. This emphasizes the need for EO probes measuring E field and H field components. The E field component may be measured using the electro-optic effect of crystals [71]–[82], the converse piezoelectric effect [83]–[86] or electrostatic attraction [87]–[90]. EO probes measure the H field component based on the Faraday effect [91], [92], magnetostriction [93]–[104] or refractive index tunability of magnetic fluid [105]–[115]. The operating principle of the individual probes was described very well by scientists in [116]. In addition, they showed a table with the parameters of various EO probes, including the detection mechanism, sensor probe configuration, detection range and sensitivity/resolution.

Figure 8 contains a schematic of the electro-optical probe that is most frequently used in QoMC sensors.

The electro-optic probe is the most common solution used in QoMC. It is based on the Pockels effect [58], where refractive index change is caused by the EM field's interaction with the crystal. This change may be observed and measured as variation in polarization. The other approach incorporates a non-metallic material (most often a  $\text{LiNbO}_3$



**Fig. 8.** Diagram describing the electro-optic detection process: lock-in amplifier (purple), spectrum analyzer (purple), and EO – electro-optic crystal (blue lines are fiber connections, dashed lines are copper wires).

crystal [116], [58]) that has very little effect on EM field distribution, but will change its physical properties when EM field is applied, used as a sensing element. The polarized laser beam is used as an information carrier, as it measures the change in physical properties of the sensor material. Either polarization or intensity variations are usually detected while using the optical spectrum analyzer. Additionally, laser light is modulated with a reference signal to achieve better sensitivity and to enable the use of a lock-in amplifier. Light is detected directly or by the Mach-Zehnder interferometer. The test setup may be optically coupled – in this case a fiber circulator is used to separate the signal coming into and leaving the EO probe crystal and a gradient index (GRIN) lens is usually used to couple the laser between the fiber and the EO crystal. Open space configurations are used as well. To provide linear polarization, the polarizer is employed at the laser output. A second polarizer is inserted just before the detector and is used to filter light that has been changed in the EO crystal. In this manner, the detecting sensor will measure different light intensities and the detected change will be proportional to the shift in polarization, based on the Pockels effect caused by the EM field.

Photoconductive antennas (PCAs) may also be used as sensors for terahertz electromagnetic fields. Such an application of those antennas has been described, *inter alia*, in [117]. There are several other probes for radiation pattern measurements [56], which are placed on a plane that is perpendicular to the radiation source. These include, among others, pyroelectric cameras [118], [119] consisting of an array of pyroelectric elements, thermal paper [120], and liquid crystal sheets [121].

## 6. Summary

This paper has reviewed the wide scope of the experimental verification methods of the gyrotron quasi-optical mode converters. The only practical option for safe and precise measurements of the QoMC is the so-called “cold test method” without the presence of high EM power and also without the necessity to provide a good quality vacuum. Using such a technique the complete system of the

resonator, taper, launcher and mirrors set can be explored, by using of precise but vulnerable measuring equipment. The key component of the cold test test bed is the higher order modes generator.

The launcher and mirrors set can be considered as a system of radiating components operating at terahertz frequency range. Therefore, for measuring EM field nearby the radiators the methods developed to perform measurements in an antenna near-field can be used. The paper presents numerous examples of such measurement carried out by various teams of researchers. The tests of field distribution in the QoMC are usually reduced to scanning of individual elements with the assumption of their appropriate location in space. The most commonly used systems allow planar and cylindrical scanning. By using transformation, spherical surface can be transformed to other defined surfaces. We believe that this method will allow for scanning the hyperboloid or paraboloid surfaces corresponding to the mirrors surface. The EM field measurements in terahertz frequencies require the use of appropriate transducers that allow to convert the EM field distribution to current or voltage.

Probes are the key element in the EM near field measurement setup. The selection of the appropriate probe type determines the measured component of the EM field and the measurements spatial resolution obtained. The paper describes various types of probes used and their advantages and limitations. A very promising solution is the application of electro-optical effects, allowing for very accurate measurement of EM field distribution. The use of an optical probes allowed to reduce spatial resolution 5 times compared to monopole coaxial probes. Some non-standard methods such as thermal paper, pyroelectric cameras or liquid crystal sheet may also be considered however these methods give usually low quality images.

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# Electromagnetic Scattering of Inhomogeneous Plane Wave by Ensemble of Cylinders

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<https://doi.org/10.26636/jtit.2020.138519>

**Abstract**—The interaction between an ensemble of cylinders and an inhomogeneous plane wave is introduced and is determined, in the present paper, through a rigorous theoretical approach. Scattered electromagnetic field generated by an indefinite number of infinite circular cylinders is analyzed by the application of the generalized vector cylinder harmonics (VCH) expansion. The exact mathematical model relied upon to represent this scenario considers the so-called complex-angle formalism reaching a superposition of vectorial cylindrical-harmonics and Foldy-Lax Multiple scattering equations (FLMSE) to account for the multi-scattering process between the cylinders. The method was validated by comparing the numerical results obtained with the use of the finite element method and a homemade Matlab code.

**Keywords**—electromagnetic scattering, inhomogeneous wave dispersion, lossy media, multi-cylinders scattering, vectorial cylindrical-harmonics.

## 1. Introduction

In the last decades, researchers were focusing on solving Maxwell's equations in order to determine the field scattered by a single or by multiple objects with a specific geometry. Geometrically, electromagnetic scattering has already been analyzed in a thorough manner. In fact, spherical, spheroidal, cylindrical, conical, and ellipsoidal objects may be found in literature, in several works devoted to canonical scattering [1]–[6]. In addition to the cases mentioned above, many other complex scenarios have been examined, such as spheres, cylinders, and axially symmetric objects [7]–[13]. Ensembles of different configurations of scatterers are also considered [14]–[17]. Moreover, different physical and chemical characteristics of materials constituting the scatterer have been analyzed [18]–[21]. Over the years, the problem of describing the interaction between the electromagnetic field and a set of cylinders has led to the development of numerous exact theoretical models that are increasingly simplified [8]–[10].

In this paper, an accurate method for showing an elliptically polarized inhomogeneous plane wave as an expansion

of vector cylindrical-harmonics (VCH) is presented. Moreover, in order to analyze the multi-scattering process, the so-called T-matrix approach [22], [23] is used, applying Foldy-Lax multiple scattering equations (FLMSEs) [24], [25] to ensure continuity of tangential components of the electromagnetic fields on the surface of each scatterer. The general representation of an electromagnetic wave as an inhomogeneous wave has continuously attracted a lot of researchers' interest. An electromagnetic wave that propagates in a lossy medium is represented by a complex wave vector with two components: a phase vector and an attenuation vector. A completely lossless medium is an ideal scenario that does not exist in nature. In fact, a wave typically propagates in a lossy medium.

In 1987, the first study related to an inhomogeneous elliptically polarized plane wave was presented by Ivlev, describing the propagation in its elemental structure and investigating the energy fallout as well [26], [27]. The first application proposed subsequently by Ivlev was on an indefinite cylinder. The Adler-Chu-Fano formulation with phase and attenuation vectors [28] was adopted in these studies, leading to meaningful and elegant results with a high degree of complexity.

The present study shows how the representation of the incident field as VCHs superposition could be reached with much greater simplicity through the use of the complex-angle [29] formulation. Also, this approach will be generalized for a scenario involving scattering caused by a cylinder immersed in a lossy medium. Our goal is to use these simplified models and to introduce a concept of losses in the propagation medium in a minimally invasive way. The results will be suitable for updating models, codes, and software presented in the literature to describe real situations with a minimal impact on the code itself. The paper provides numerical comparisons of different phenomena involving cylindrical vector waves. In addition, the study focuses on the analysis of an ensemble of infinite lossy cylinders immersed in a lossy medium, and on its results. A multi-paradigm numerical computing environment (Matlab) was used to implement the various formulations, while the proper model was simulated with the

use of Comsol Multiphysics – commercial software based on FEM.

The article is divided into 4 parts. Section 2 introduces two principal formalisms required to properly represent an inhomogeneous wave and illustrates the formulas needed to switch between both of them. Subsequently, theoretical aspects are described in order to obtain a representation of an inhomogeneous elliptically polarized electric field as a superposition of VCHs. Section 3 shows numerical validations concerning results obtained in Matlab and the ones generated with the use of Comsol Multiphysics. In the same section, new results are also presented for the scattering of an elliptically polarized plane wave at oblique incidence from  $n$ -cylinders in a lossy medium. Finally, in Section 4, conclusions are given, along with potential future developments.

## 2. Theoretical Approach

From literature, two formalisms are known to be used for representing an inhomogeneous wave propagating in a lossy medium. The first one, characterized by better features as well, is the formalism known as Adler-Chu-Fano formulation. Its propagation vector has a complex nature with  $\mathbf{k}_i = \boldsymbol{\beta}_i + i\boldsymbol{\alpha}_i$  represented by phase and attenuation vectors  $\boldsymbol{\beta}_i, \boldsymbol{\alpha}_i \in \Re$ , respectively. The other one has a complex propagation vector as well, represented by the superposition of real and imaginary parts  $\mathbf{k}_i = \mathbf{k}_R + i\mathbf{k}_I$ . This vector forms a complex angle with the axis of the Cartesian reference system  $\bar{\vartheta}_i = \vartheta_R + i\vartheta_I$  [29], see Fig. 1. The symbol  $\bar{\vartheta}$  was used to underline the complex nature of the angle.

This study shows that a field expressed as a superposition of basic cylindrical waves through the use of the complex-angle formalism may be represented with relative simplicity. The following wave, in which vectors  $\alpha_i$  and  $\beta_i$  are creating angles  $\zeta_i$  and  $\eta_i$  in relation to the  $z$  axis, are also placed on the same plane passing through the  $z$  axis, and are forming a real angle  $\varphi$  with the  $x$  axis (see Fig. 1). In

this particular case, the following relations exist between the two formalisms [29]:

$$\cos \vartheta_R = \frac{k_R \beta \cos \xi + k_I \alpha \cos \eta}{\sqrt{k_R^2 \beta^2 - k_I^2 \alpha^2 + 2(k_R k_I)^2}}, \quad (1)$$

$$\sin \vartheta_R = \frac{k_R \beta \sin \xi + k_I \alpha \sin \eta}{\sqrt{k_R^2 \beta^2 - k_I^2 \alpha^2 + 2(k_R k_I)^2}}, \quad (2)$$

$$\vartheta_I = \frac{1}{2} \operatorname{atanh} \left( \frac{2\beta\alpha}{k^2} \right), \quad (3)$$

where  $\eta$  and  $\xi$  are the angles that the vectors  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$ , respectively, form with the  $z$  axis. Equations (1) and (2) play a fundamental role in assigning a value to  $\vartheta_R$  avoiding its indetermination. Plane  $\varphi = 0$  was considered due to its simplicity, although the following considerations can be easily expanded to each plane with  $\varphi \neq 0$ .

The solution of the scalar Helmholtz equation provides the following scalar formula [30]–[36]:

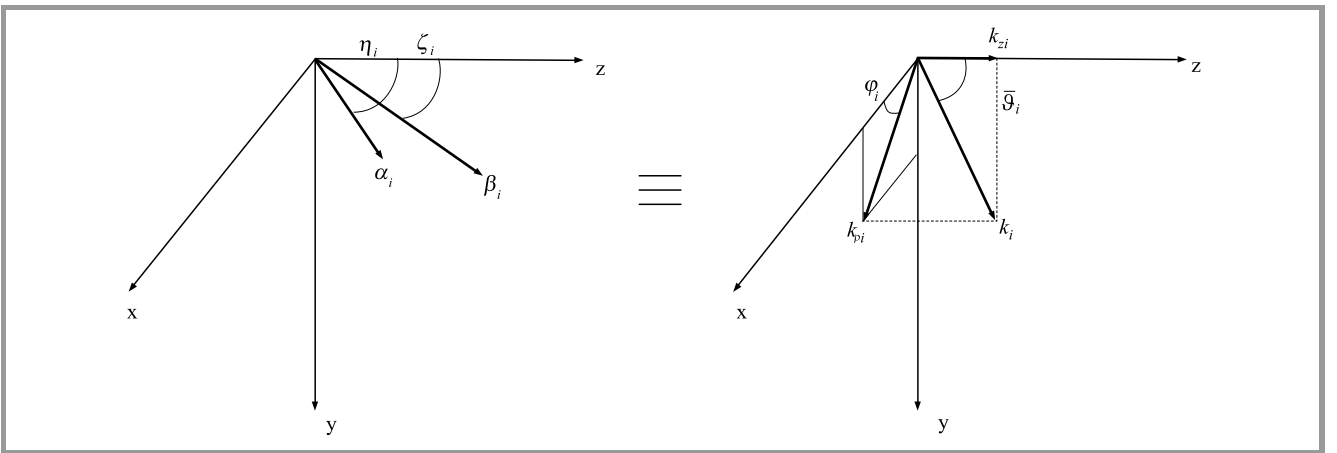
$$\psi_m = A e^{im\varphi} Z_m(k_\rho \rho) e^{ik_z z - i\omega t}, \quad (4)$$

where  $\rho, \phi, z$  are three variables independent of the cylindrical coordinate system,  $A$  is a complex constant, while  $k_\rho$  and  $k_z$  are the projections of the propagation vector on the plane  $z = 0$  and the on the  $z$  axis, respectively. The last two components are defined as shown below:

$$k_\rho^2 + k_z^2 = k^2, \quad (5)$$

with  $k_x = k_\rho \cos \varphi$  and  $k_y = k_\rho \sin \varphi$  being projections of the transversal vector  $k_\rho$  on the plane  $z = 0$ . The function  $Z_m(k_\rho \rho)$  describes the first, second, third, and fourth Bessel functions as  $J_m(k_\rho \rho)$ ,  $Y_m(k_\rho \rho)$ ,  $H_m^{(1)}(k_\rho \rho)$ , and  $H_m^{(2)}(k_\rho \rho)$ , respectively. Hence, the harmonic vector is characterized as [31]–[36]:

$$\mathbf{M} = \nabla \times (\hat{\mathbf{z}}_0 \psi), \quad \mathbf{N} = \frac{1}{k} \nabla \times \mathbf{M}. \quad (6)$$



**Fig. 1.** The figure on the left represents the complex wave vector of an inhomogeneous plane wave with phase and attenuation vectors. The figure on the right represents the same vectors with the complex-angle formulation.

It is always possible to define electric and the magnetic fields as a superposition of these vectorial functions:

$$\mathbf{E} = \sum_{m=-\infty}^{+\infty} (a_m \mathbf{M}_m + b_m \mathbf{N}_m), \quad (7)$$

$$\mathbf{H} = \frac{k}{i\omega\mu} \sum_{m=-\infty}^{+\infty} (a_m \mathbf{N}_m + b_m \mathbf{M}_m). \quad (8)$$

Let us consider a simple inhomogeneous plane wave, using the formalism presented for the first time by Frezza *et al.* [36], [37]. Any obliquely polarized elliptical field, with respect to the surface of a cylinder, can be represented as a linear combination of two components, one vertical ( $\mathbf{v}_0$ ) and one horizontal ( $\mathbf{h}_0$ ), each multiplied by its polarization coefficient ( $E_{vi}$  and  $E_{hi}$ , respectively):

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= [E_{vi} \mathbf{v}_0(\bar{\vartheta}_i, \varphi_i) + E_{hi} \mathbf{h}_0(\bar{\vartheta}_i, \varphi_i)] e^{i\mathbf{k} \cdot \mathbf{r}} \\ &= \sum_{m=-\infty}^{+\infty} [a_m \mathbf{M}_m(k^* \mathbf{r}) + b_m \mathbf{N}_m(k^* \mathbf{r})], \end{aligned} \quad (9)$$

imposing the following definitions [36]:

$$a_m = \frac{E_{hi}}{k_p} (-i)^{m-1} e^{-im\varphi_i}, \quad (10)$$

$$b_m = -\frac{E_{vi}}{k_p} (-i)^m e^{-im\varphi_i}, \quad (11)$$

$$\mathbf{k}_i = k^* (\sin \bar{\vartheta}_i \cos \varphi_i \mathbf{x}_0 + \sin \bar{\vartheta}_i \varphi_i \mathbf{y}_0 + \cos \bar{\vartheta}_i \mathbf{z}_0), \quad (12)$$

$$\mathbf{M}_m = \mathbf{m}_m e^{im\varphi} e^{ik_z z - i\omega t}, \quad (13)$$

$$\mathbf{N}_m = \mathbf{n}_m e^{im\varphi} e^{ik_z z - i\omega t} \quad (14)$$

with:

$$\mathbf{m}_m = im \frac{Z_m(k_p \rho)}{\rho} \boldsymbol{\rho}_0 - k_p \frac{\partial \rho Z_m(k_p \rho)}{\partial \rho} \boldsymbol{\varphi}_0, \quad (15)$$

$$\begin{aligned} \mathbf{n}_m &= i \frac{k_z k_p}{k} \frac{\partial \rho Z_m(k_p \rho)}{\partial \rho} \boldsymbol{\rho}_0 - \frac{mk_z}{k} \frac{Z_m(k_p \rho)}{\rho} \boldsymbol{\varphi}_0 + \\ &+ \frac{k_p^2}{k} Z_m(k_p \rho) \mathbf{z}_0, \end{aligned} \quad (16)$$

having indicated, by  $k^*$ , the complex conjugate of the wavenumber  $k$  and by  $\boldsymbol{\rho}_0$ ,  $\boldsymbol{\varphi}_0$  the unit vectors of the cylindrical coordinate system. Considering several parallel cylinders in free space, let us analyze their scattering with the defined incident field, as shown in Fig. 2. An arbitrarily assigned number  $L$  of dielectric cylinders, with relative permittivities  $\epsilon_j$ , with  $j = 1, \dots, N$ , infinite length and radii  $r_j$  in a free-space filled by a lossy medium, in general dissipative, with relative permittivity  $\epsilon_e$ , relative permeability  $\mu_e$ , and electric conductivity  $\sigma_e$  is considered. The incident field, as usual, is an elliptically polarized inhomogeneous plane wave. In order to apply Foldy-Lax multiple scattering equations, the external field on the surface of the  $q$ -th cylinder, also referred to as the exciting field, needs to be taken into consideration. The exciting

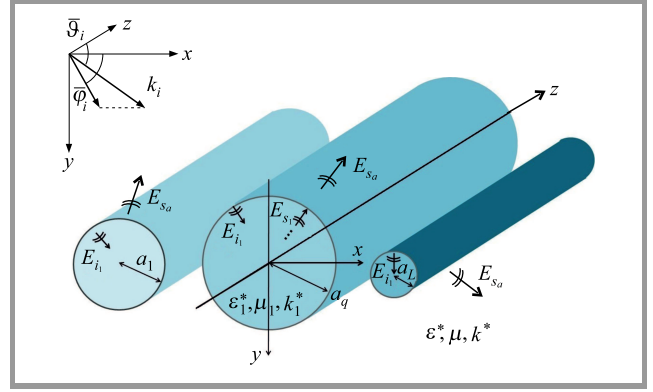


Fig. 2. Depiction of the problem.

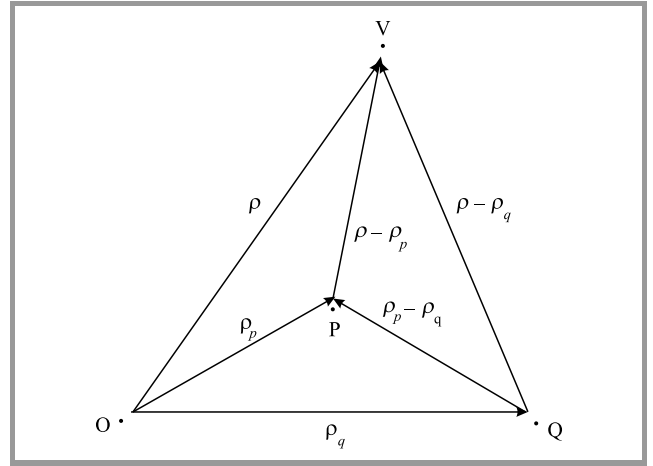


Fig. 3. Given three reference systems (O, P, Q), the following figure shows the reciprocal directions of vectors  $\boldsymbol{\rho}$  with respect to an external observation point V.

field ( $\mathbf{E}_{ex}^q$ ) is the superposition of the incident field ( $\mathbf{E}_i$ ) and all fields scattered by the cylinders ( $\mathbf{E}_s^p$ ), see Fig. 3:

$$\mathbf{E}_{ex}^q = \mathbf{E}_i + \sum_{\substack{p=1 \\ p \neq q}}^L \mathbf{E}_s^p. \quad (17)$$

The incident field may be expressed as a function of vector cylindrical harmonics centered on the  $q$ -th cylinder [36]:

$$\begin{aligned} \mathbf{E}_i(k \boldsymbol{\rho}_q) &= [E_{vi} \mathbf{v} + E_{hi} \mathbf{h}] e^{i\mathbf{k} \cdot \boldsymbol{\rho}_q} e^{i\mathbf{k} \cdot (\boldsymbol{\rho} - \boldsymbol{\rho}_q)} = \\ &= \sum_{m=-\infty}^{+\infty} [\tilde{a}_m \mathbf{M}_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) + \tilde{b}_m \mathbf{N}_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q)], \end{aligned} \quad (18)$$

with:

$$\tilde{a}_m = a_m e^{i\mathbf{k} \cdot \boldsymbol{\rho}_q}, \quad \tilde{b}_m = b_m e^{i\mathbf{k} \cdot \boldsymbol{\rho}_q} \quad (19)$$

with  $k$  indicating the wavenumber of the host medium (see Fig. 2) and with  $\boldsymbol{\rho}_q$  indicating the radial direction centered in the  $q$ -th cylinder (see Fig. 3). The exiting field of the  $q$ -th cylinder is:

$$\mathbf{E}_{ex}^q(k\boldsymbol{\rho}_q) = \sum_{m=-\infty}^{+\infty} \left[ w_m^q \mathbf{M}_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) + v_m^q \mathbf{N}_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) \right], \quad (20)$$

while the scattered electric field from  $p \neq q$ -th cylinder is:

$$\mathbf{E}_s^p(k\boldsymbol{\rho}_p) = \sum_{m'=-\infty}^{+\infty} \left[ T_{m'}^M w_{m'}^p \mathbf{M}_{m'}^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_p) + T_{m'}^N v_{m'}^p \mathbf{N}_{m'}^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_p) \right], \quad (21)$$

with  $T_{m'}^M$  and  $T_{m'}^N$  indicating the scattering coefficients in the dielectric cylinder case, i.e. the T-matrix coefficients [24], [25]:

$$T_m^M = -\frac{J'_m(k_{ip}a)}{H'_m^{(1)}(k_{ip}a)}, \quad (22)$$

$$T_m^N = -\frac{J_m(k_{ip}a)}{H_m^{(1)}(k_{ip}a)}, \quad (23)$$

while coefficients  $w_{m'}^p$  and  $v_{m'}^p$  represent the unknowns of our problem. By applying the addition theorem to the VCHs function, we obtain:

$$M_{m'}^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) = \sum_{m=-\infty}^{+\infty} A_{mm'} M_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q), \quad (24)$$

$$N_{m'}^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) = \sum_{m=-\infty}^{+\infty} A_{mm'} N_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q), \quad (25)$$

$$M_{m'}^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) = \sum_{m=-\infty}^{+\infty} B_{mm'} M_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q), \quad (26)$$

$$N_{m'}^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) = \sum_{m=-\infty}^{+\infty} B_{mm'} N_m^{(1)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q), \quad (27)$$

with:

$$A_{mm'} = H_{m-m'}^{(1)}(k|\boldsymbol{\rho}_p - \boldsymbol{\rho}_q|) e^{-i(m-m')\varphi_{pq}}, \quad (28)$$

$$B_{mm'} = J_{m-m'}^{(1)}(k|\boldsymbol{\rho}_p - \boldsymbol{\rho}_q|) e^{-i(m-m')\varphi_{pq}}. \quad (29)$$

By replacing all fields inside the FLMSEs and using the orthogonal properties of the VCHs, the following linear system is obtained:

$$w_m^q = \tilde{a}_m + \sum_{m'=-\infty}^{+\infty} \sum_{\substack{p=1 \\ p \neq q}} A_{mm'} T_{m'}^M w_{m'}^p, \quad (30)$$

$$v_m^q = \tilde{b}_m + \sum_{m'=-\infty}^{+\infty} \sum_{\substack{p=1 \\ p \neq q}} A_{mm'} T_{m'}^N v_{m'}^p. \quad (31)$$

At this point, the linear system may be solved and coefficients  $w_m^q$  and  $v_m^q$  may be determined. With the field scattered by the  $q$ -th cylinder, writable as a superposition of VCHs, as:

$$\mathbf{E}_s^q = \sum_{m=-\infty}^{+\infty} \left[ e_m^q \mathbf{M}_m^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) + f_m^q \mathbf{N}_m^{(3)}(k, \boldsymbol{\rho} - \boldsymbol{\rho}_q) \right], \quad (32)$$

the coefficients of the  $q$ -th cylinder may be written as follows:

$$e_m^q = T_m^M w_m^q, \quad (33)$$

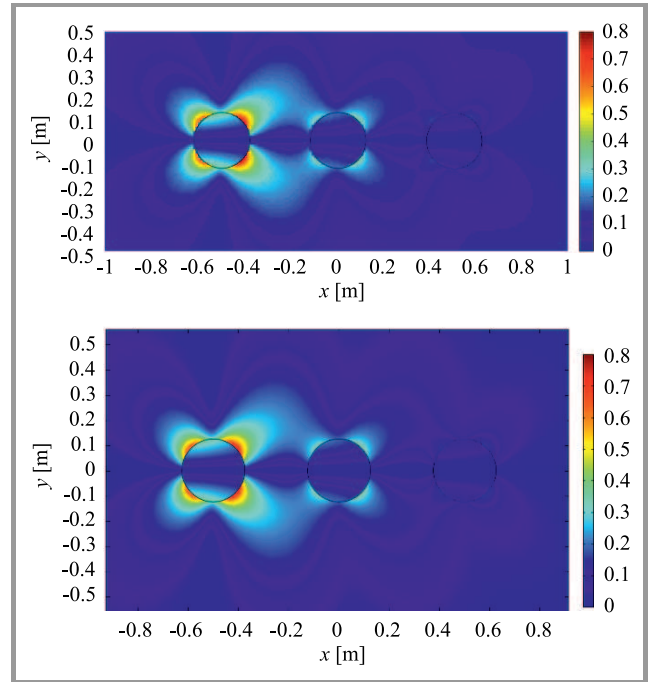
$$f_m^q = T_m^N v_m^q, \quad (34)$$

and the total scattered field may be obtained:

$$\mathbf{E}_s = \sum_{q=1}^L \mathbf{E}_s^q. \quad (35)$$

### 3. Validation and Numerical Results

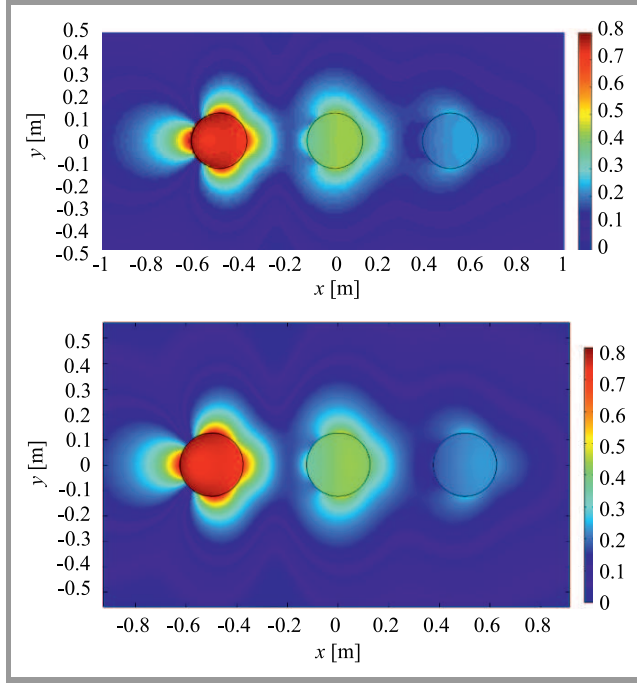
Results of the comparison between the formulation and the canonical case of electromagnetic scattering were validated. In this research paper, three infinite lossy dielectric cylinders have been considered with a radius of  $a_1 = a_2 = a_3 = 0.125$  m. The three cylinders are centered at the following  $z$  plane coordinates:  $C_1 = [-0.5, 0]$ ,  $C_2 = [0, 0]$ , and  $C_3 = [0.5, 0]$  m. A 1 V/m plane wave with a frequency of 300 MHz is impinging on these cylinders.



**Fig. 4.** Comparison of spatial distribution (on the  $x$ - $y$  plane) of the absolute value of the  $x$  component of the scattered electric field  $|E_{sx}|$ . In particular, Matlab (top) and Comsol (bottom) results are shown. (For color pictures see the electronic version of the paper).

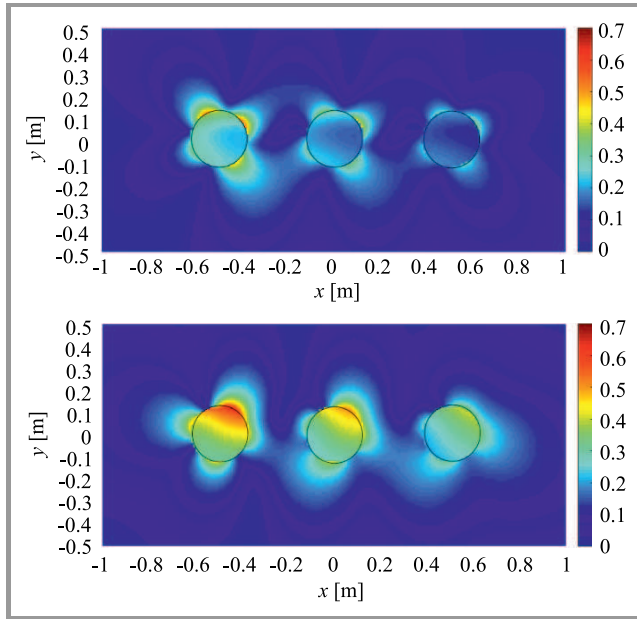
In Figs. 4 and 5, results obtained with Matlab and Comsol in the case of  $k_e = 1 - i/10$  for the environment and  $k_c = 2 - 0.5i/10$  for all cylinders (arbitrary parameters) for an incident wave directed at  $\vartheta_i = \pi/2$  and  $\varphi_i = 0$  rad are displayed. In particular, the absolute values of the  $x$  and  $y$  components of the scattered electric field (the  $z$  component is null) are shown. As noted, the results are perfectly compatible.





**Fig. 5.** Comparison of spatial distribution (on the  $x$ - $y$  plane) of the absolute value of the  $y$  component of the scattered electric field  $|E_{sy}|$ . In particular, Matlab (top) and Comsol (bottom) results are shown.

Subsequently (see Fig. 6), the most general case of an inhomogeneous plane wave with  $\vartheta$  complex value was shown. In particular, we have considered the following values for the complex angle:  $\vartheta = \pi/6(1 + 0.5i)$  rad and  $\varphi = \pi/4$  rad (arbitrary parameters). The remaining parameters have remained the same as in the previous case. To obtain these



**Fig. 6.** Spatial distribution of the absolute value (on the  $x$ - $y$  plane) of the  $x$  (top) and  $y$  (bottom) components of the scattered electric field  $|E_{sx}|$ ,  $|E_{sy}|$  obtained with Matlab.

results,  $N = 5$  was used as the number of terms for the VCHs series [38].

In Fig. 6, the contribution of the complex angle is highlighted by the direction of the scattered wave which creates an attenuation effect spreading in the same direction.

## 4. Conclusions

An accurate method allowing to express an inhomogeneous, elliptically polarized plane wave in terms of vectorial cylinder harmonics, needed to solve the problem of multi-scattering generated by an ensemble of cylinders, is presented. Determination of the expansion coefficients and application of the so-called Foldy-Lay equations required to use the complex-angle formalism contribute to the determination of the solution to the electromagnetic problem. A light and elegant formalism has been reached thanks to such an approach. The procedure was validated with some numerical results as well as with comparisons performed via simulations in the COMSOL environment. In particular, the results were compared with regard to scattering created by three lossy dielectric cylinders of a circular cross-section and infinite length. Perfect compatibility was reached in all scenarios. The cylindrical harmonics defined with the complex angle share the same properties as simple cylindrical harmonics, thanks to the minimal invasiveness of the formalism.

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(Contents Continued from Front Cover)

**Review of Experimental Verification Methods of Gyrotron  
Quasi-optical Mode Converters**

*G. Jaworski, A. Francik, M. Nowak, and K. Nowak*

*Paper*

75

**Electromagnetic Scattering of an Inhomogeneous Plane Wave  
by Ensemble of Cylinders**

*L. Dinia, F. Mangini, and F. Frezza*

*Paper*

86



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