Performance Analysis of Hybrid Phase Shift Keying over Generalized Nakagami Fading Channels

Mahmoud Youssuf and Mohamed Z. Abdelmageed

Abstract—In addition to the benefits of hybrid phase shift keying (HPSK) modulation in reducing the peak to average power ratio of the transmitted signal to reduce the zero crossings and the 0°-degree phase transmissions, HPSK enhances the bit error rate (BER) measure of the signal performance. The constellation of the HPSK is analyzed, and an expression for the conditional probability of HPSK modulation over additive white Gaussian noise (AWGN) is derived. This BER measure of HPSK is shown to outperform quadrature phase shift keying (QPSK) modulation. HPSK performance through Nakagami – m fading channel is also considered.

Keywords—bit error rate, hybrid phase shift keying, quadrature phase shift keying.

1. Introduction

Hybrid phase shift keying (HPSK) is used in wideband code division multiple access (WCDMA) systems thanks to its low peak-to-average power ratio. This low ratio of peak-to-average power results in reducing the number of zero crossing of phase transitions of the output transmitted signal. In this paper we prove that HPSK outperforms other quadrature modulation techniques, such as offset quadrature phase shift keying (OQPSK) by 3 dB.

The paper is organized as follows: Section 2 describes the HPSK constellation in case of two channels at different amplitudes. Section 3 derives an expression for the conditional probability of error of HPSK modulated signal over an additive white Gaussian noise (AWGN). In Section 4 we apply the obtained expression in evaluating the performance of HPSK modulated signal over a generalized Nakagami – m channel. Finally, Section 5 includes numerical results and comments.

2. The Hybrid Phase Shift Keying Constellation

The HPSK has been proposed as the spreading technique for WCDMA to eliminate the zero crossings for every other signal transition and to eliminate the 0°-degree phase shift transitions for every other chip point, as well as to improve the bit error rate (BER) measure of the direct sequence WCDMA (DS-WCDMA) system performance.

In 3G systems the mobile station (MS) can transmit more than one channel. The different channels are assigned to either I or Q path.

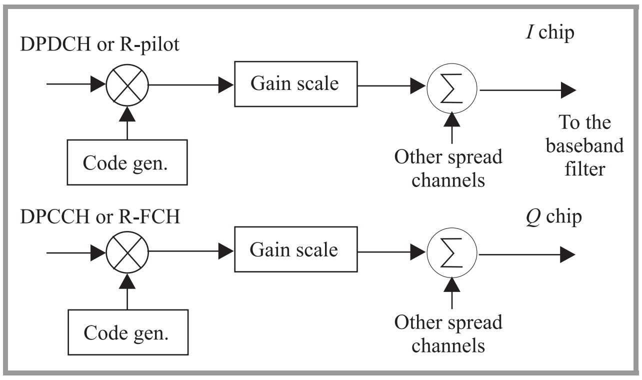

Fig. 1. The basic reverse channel structure of 3G system. Explanations: DPCCH – dedicated physical control channel, DPDCH – dedicated physical data channel, R-FCH – reverse fundamental channel.

In the case of transmitting only two channels, as in Fig. 1, one of the channels (DPDCH or R-pilot) is applied to the I path and the other channel (DPCCH or R-FCH) is applied to the Q path [1]. Additional, high data rate channels are combined alternatively on the I or Q paths. Each channel is spread by a different orthogonal even-numbered Walsh code. In the general case the channels can be at

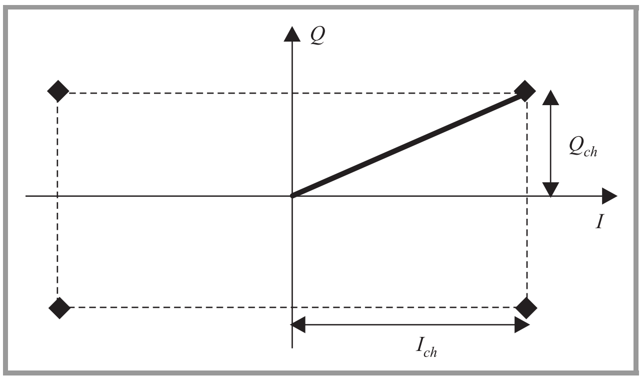

Fig. 2. The 4-QAM constellation for two channels at different amplitudes.

different power levels, as in Fig. 2 which maps onto a rectangular four quadrature amplitude modulation (4-QAM) constellation.

The QAM signal waveforms may be expressed as

$$S_m(t) = \mathrm{Re}\left\{A e^{j\theta_m} g(t) e^{j2\pi f_c t}\right\} = A_{ch} g(t)\left[\cos(2\pi f_c t)\cos(\theta_m) - \sin(2\pi f_c t)\sin(\theta_m)\right], \quad (1)$$

where: $A_{ch} = \sqrt{I_{ch}^2 + Q_{ch}^2}$, where I_{ch} and Q_{ch} are the information bearing signals amplitudes of the I path channel and Q path channel, respectively, $g(t)$ is the pulse shape signal, $\theta_m = \tan^{-1}\frac{Q_{ch}}{I_{ch}}$ is the phase of the signal vector and it varies with $m = 1, 2, 3, 4$.

So, the QAM signal waveform may be viewed as a linear combination of two orthogonal wave forms $f_1(t)$ and $f_2(t)$ such that [2]:

$$S_m(t) = S_{m1} f_1(t) + S_{m2} f_2(t), \quad (2)$$

where S_{m1} and S_{m2} are the component of the two dimensional vector S_m:

$$S_m = \left[S_{m1} \quad S_{m2}\right] = \left[I_{ch} \quad Q_{ch}\right]. \quad (3)$$

In the 4-QAM according to the position of the vector point and according to the value of I_{ch} and Q_{ch}:

$$\therefore S_m = \left[A_{ch}\cos\theta_m \quad A_{ch}\sin\theta_m\right]. \quad (4)$$

In the reverse link of DS-CDMA systems the I_{ch} and Q_{ch} are complex scrambled with a complex scrambling signal $(I_s + jQ_s)$ as in Fig. 3.

The final I and Q signals are produced mathematically by the multiplication of the two complex signals; the complex data signal $(I_D + jQ_D)$ which has already spread into chips $(I_{ch} + jQ_{ch})$, and the complex scrambling signal $(I_s + jQ_s)$ so the final I and Q signals are:

$$I + jQ = (I_{ch}I_s - Q_{ch}Q_s) + j(I_{ch}Q_s + Q_{ch}I_s) = A_{ch}A_s e^{j(\phi_{ch}+\phi_s)},$$

$$I = A_{ch}A_s\cos\left(\frac{\pi}{M}(2n-1) + \theta_m\right) = A\cos\left(\frac{\pi}{M}(2n-1) + \theta_m\right), \quad (5)$$

$$Q = A_{ch}A_s\sin\left(\frac{\pi}{M}(2n-1) + \theta_m\right) = A\sin\left(\frac{\pi}{M}(2n-1) + \theta_m\right). \quad (6)$$

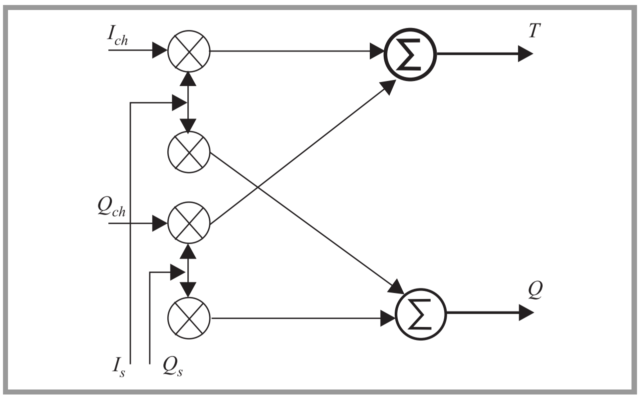

Fig. 3. Complex scrambling of HPSK.

Since final constellation is formed by complex multiplication of the two signals of chip constellation and scrambling constellation which is always QPSK constellation as in Fig. 4, then: from Eqs. (5) and (6) we conclude that the final signal $(I + jQ)$ is a QPSK constellation with two dimensional vector representation S_{mn}, where:

$$S_{mn} = \left[A\cos\left(\frac{\pi}{M}(2n-1) + \theta_m\right) \quad A\sin\left(\frac{\pi}{M}(2n-1) + \theta_m\right)\right], \quad (7)$$

where $A = A_{ch}A_s$; $n = 1, 2, 3, 4$; $m = 1, 2, 3, 4$ and $M = 4$.

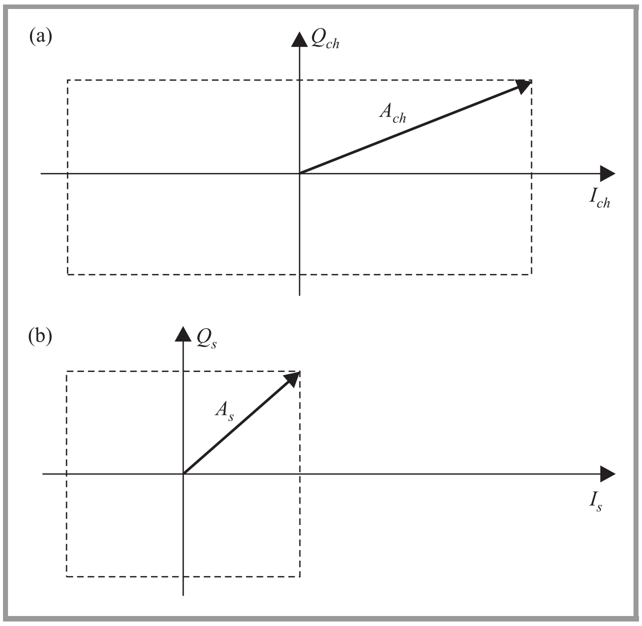

Fig. 4. The chip constellation (a) and the scrambling constellation (b).

This new constellation has points that rotate according to the angle: $\frac{\pi}{4}(2n-1) + \theta_m$, while θ_m changes according to the value of I_{ch} and Q_{ch}, for example, if $n = 1$, the point (I_{ch}, Q_{ch}) will be transferred by the angle equal to $(45° + \theta_m)$. So the new constellation is really an eight point constellation with two independent QPSK constellation as shown in Fig. 5 according to the value of θ_m. One of these two constellations corresponds to $\theta_m > 45°$ and it rotates by angle equal to $\phi_1 = (\theta_m - 45°)$ from the original axes. The other QPSK constellation corresponds to $\theta_m < 45°$ and rotates with angle equal to $\phi_2 = -(\theta_m - 45°)$ from the original axes. So the new final constellation consists of two independent QPSK constellations with complex scrambling:

$$S_n = \left[A\cos\left[\frac{2\pi}{M}(n-1) + \phi_1\right] \quad A\sin\left[\frac{2\pi}{M}(n-1) + \phi_1\right]\right], \quad (8)$$

$$S'_n = \left[A\cos\left[\frac{2\pi}{M}(n-1) + \phi_2\right] \quad A\sin\left[\frac{2\pi}{M}(n-1) + \phi_2\right]\right]. \quad (9)$$

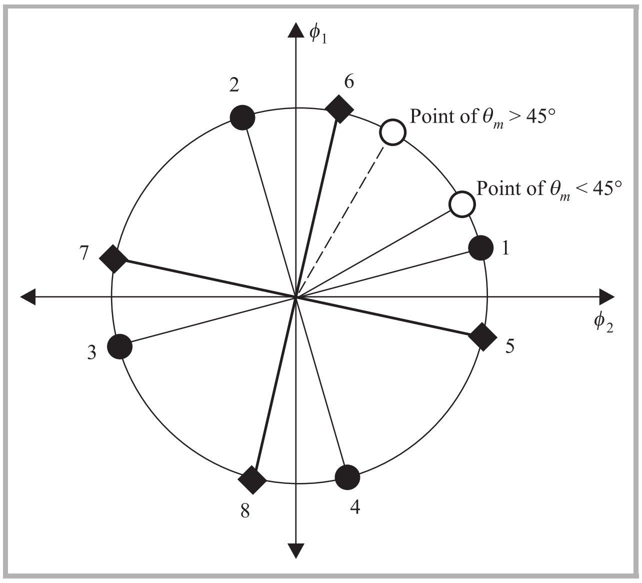

Fig. 5. The final constellation of the scrambled chip of different channel amplitudes.

So, the final constellation has eight points, as in Fig. 5, with the angular distribution determined by the relative levels of the two channels signals.

3. Probability of Error of Hybrid Phase Shift Keying Modulated Signal over Additive White Gaussian Noise

We concluded in the previous section that the final constellation of the two channels at different amplitudes consists of two independent QPSK constellations with complex scrambling. The average value of the amplitude of the new constellation is $\sqrt{2}$ times the value of the amplitude of the traditional QPSK. To obtain the BER of the HPSK modulated signal as a measure of its performance we will assume that this signal is transmitted over AWGN channel. The received signal is demodulated with correlated demodulator or a matched filter demodulator and the decision for the received observation vector $r = [r_1, r_2, r_3, \ldots, r_N]$ among all the transmitted signals S_m is based on the maximum of the conditional probability distribution function (pdf) $P(r/S_m)$, which is the maximum likelihood (ML).

The optimum ML detector computes a set of M correlation metrics $C(r, S_m) = -2rS_m$ and selects the signal corresponding to the largest correlation metric [3].

Applying this metric in our study case, r is the received signal vector $r = [r_1 \;\; r_2]$ which is projected onto each of the four possible transmitted signals vectors S_m for $m = 1, 2, 3, 4,$ where $M = 4$. We can consider that the correlation detector in the case of HPSK modulated signal is equivalent to a phase detector that computes the phase of the received signal from r and selects the signal S_m whose phase is closest to r, where the phase of r is $\theta_r = \tan^{-1}\frac{r_2}{r_1}$.

The probability of error can be computed if we determine the power density function of θ_r $P_{\theta_r}(r)$. We consider the case in which the transmitted signal phase is equal to zero, as in Fig. 6.

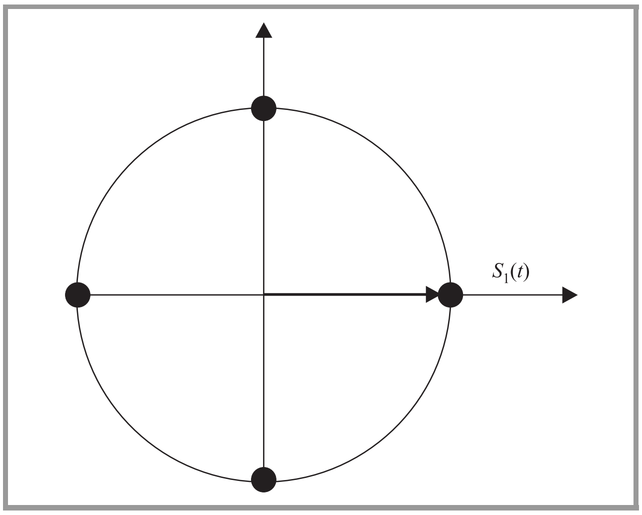

Fig. 6. The HPSK vector constellation.

The transmitted vector $S_1 = \left[\sqrt{\varepsilon_s} \;\; 0\right]$, where ε_s is the energy of the transmitted HPSK signal $S_1(t)$. The received signal vector is $r = [r_1 \;\; r_2] = \left[\sqrt{\varepsilon_s} + n_1 \;\; n_2\right]$, where n_1 and n_2 are jointly Gaussian random variables with mean and variances $E(r_1) = \sqrt{\varepsilon_s}$, $E(r_2) = 0$ and $\sigma r_{12} = \sigma r_{22} = \sigma r_2 = \frac{1}{2}N_0$. Consequently, the joint pdf of r_1 and r_2 is:

$$P(r_1, r_2) = \left(\frac{1}{\pi N_0}\right)\exp\left[-\frac{(r_1 - \sqrt{\varepsilon_s})^2 + r_2^2}{N_0}\right]. \quad (10)$$

The pdf of the phase θ_r is obtained by a change in variables from (r_1, r_2) to (A, θ_r), where:

$$A = \sqrt{r_1^2 + r_2^2}, \quad \theta_r = \tan^{-1}\frac{r_2}{r_1},$$

$$P(A, \theta_r) = A\left(\frac{1}{\pi N_0}\right)\exp\left[-\frac{(A^2 + \varepsilon_s - 2\sqrt{\varepsilon_s}A\cos\theta_r)}{N_0}\right], \quad (11)$$

$$P_{\theta_r}(\theta_r) = \int_0^\infty P(A, \theta_r)\mathrm{d}A = \frac{1}{2\pi}\mathrm{e}^{-2\gamma_s\sin^2\theta}\int_0^\infty A\mathrm{e}^{-\frac{A-\sqrt{4\gamma_s}\cos\theta)^2}{2}}, \quad (12)$$

where $\gamma_s = \frac{\varepsilon_s}{N_0}$ is the signal-to-noise ratio (SNR).

For large values of $\gamma_s \gg 1$ and $|\theta_r| \leq \frac{\pi}{2}$, $P_{\theta_r}(\theta_r)$ is well approximated as

$$P_{\theta_r}(\theta_r) = \frac{1}{2\pi} e^{-2\gamma_s \sin^2\theta_r} I\,, \tag{13}$$

where

$$I = \int_0^\infty A \exp\left(-\frac{(A-\cos\theta_r\sqrt{4\gamma_s})^2}{2}\right) \mathrm{d}A\,, \tag{14}$$

$$I = \cos\theta_r \sqrt{2\pi}\sqrt{4\gamma_s}\,, \tag{15}$$

$$P_{\theta_r}(\theta_r) = \sqrt{\frac{2\gamma_s}{\pi}} \cos\theta_r \mathrm{e}^{-2\gamma_s \sin^2\theta_r}\,. \tag{16}$$

When $s_1(t)$ is transmitted a decision of error is made if the noise causes the phase to fall outside the range $\left[-\frac{\pi}{4},\, \frac{\pi}{4}\right]$, and hence the probability of a symbol error is:

$$P_4 = 1 - \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} P_{\theta_r}(\theta_r) \mathrm{d}\theta_r\,. \tag{17}$$

By substituting for $P_{\theta_r}(\theta_r)$ and performing the change of variables from θ_r to μ, where $\mu = \sin\theta_r\sqrt{2\gamma_s}$, we find

$$P_4 = 2Q\left(\sqrt{2\gamma_s \sin^2\left(\frac{\pi}{4}\right)}\right) = 2Q\left(\sqrt{\gamma_s}\right). \tag{18}$$

In case of HPSK

$$\varepsilon_s = 2(k\varepsilon_b) = 4\varepsilon_b \;\;\therefore P_4 = 2Q\left(\sqrt{\frac{4\varepsilon_b}{N_0}}\right). \tag{19}$$

Since the transmitted signals represented by the vector points of the final constellation are equally likely to be transmitted and since the 8-points are distributed around a circle consisting of two independent QPSK constellations with complex scrambling then the average probability of the symbol error in the case of two channels at different amplitudes is:

$$P_{symbol} = \frac{1}{2}\left[P_1(e) + P_2(e)\right], \tag{20}$$

where $P_1(e)$ is the probability of symbol error of the first QPSK constellation, $P_2(e)$ is the probability of symbol error of the second constellation:

$$P_S = 2Q\left(\sqrt{\frac{4\varepsilon_b}{N_0}}\right). \tag{21}$$

The bit error probability in this case is:

$$P_b = Q\left(\sqrt{\frac{4\varepsilon_b}{N_0}}\right) = Q\left(\sqrt{4\gamma}\right), \tag{22}$$

where γ is the SNR $= \varepsilon_b/N_0$.

It is simply interesting to compare the performance of HPSK with that of QPSK since both types of signals are two dimensional. Since the error probability is dominant by the arguments of the Q function, we may simply compare the arguments of Q for the two signal formats of HPSK and QPSK. The ratio of the two arguments is equal $R = 2$. So, HPSK has SNR advantage of 10 log 2 $\approx$ (3 dB) over QPSK.

4. The Performance of HPSK Signal over a Generalized Nakagami – m Channel

The mobile communication channel is noisy, multipath and is subjected to fading. The channel fading conditions depend on the propagation conditions and the clutter types the waves propagate through. In some cases the fading can be more severe than Rayleigh, while in other cases where line of sight or near line of sight conditions is available, the signal will be more stable. The Nakagami distribution is shown to fit results more generally than other distributions [4]. In this section we will evaluate the average BER of HPSK systems subjected to Nakagami fading. We will evaluate the expected value of the conditional P_b as given by Eq. (22) over Nakagami distribution. In Nakagami channel the path amplitude probability density function is given by

$$f_r(r) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m r^{2m-1} \exp\left(-\frac{m}{\Omega}r^2\right), \tag{23}$$

where m is the fading parameter and it describes the fading severity which is defined as the ratio of moments or it is the ratio of the square of the mean signal power to the variance of the signal power

$$m = \frac{\Omega^2}{E\left[r^2 - \Omega\right]}\,, \quad m \geq 0.5, \quad \Omega = E\left[r^2\right]. \tag{24}$$

The received signal power γ follows gamma distribution and its pdf is given by

$$f_r(\gamma) = \left(\frac{m}{\Omega}\right)^m \frac{\gamma^{\,m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\Omega}\gamma\right); \quad \gamma \geq 0,\; m \geq 0.5\,.$$

The average probability of error will be given by

$$P(e) = \int_0^\infty P(e/\gamma) f_\gamma(\gamma)\mathrm{d}\gamma = \int_0^\infty Q\left(\sqrt{4\gamma}\right) f_\gamma(\gamma)\mathrm{d}\gamma. \tag{25}$$

The integral of the average probability is evaluated in [5]. The average probability of error is finally given by

$$P_e = \left(\frac{1}{2\sqrt{\pi}}\right) \frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m+1)} \left(\frac{2m}{2m+4\Omega}\right)^m \times {}_2F_1\left(\frac{1}{2},\, m;\; m+1;\; \frac{2m}{2m+4\Omega}\right), \tag{26}$$

where ${}_2F_1(a, b;\; c, x)$ is the Gauss hypergeometric function [6] defined by

$${}_2F_1(a, b;\, c, x) = \sum_{n=0}^\infty \frac{(a)_n(b)_n}{(c)_n} \frac{x^n}{\Gamma(n+1)} \tag{27}$$

or with integral representation:

$${}_2F_1(a, b;\, c, x) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-xt)^{-a}\mathrm{d}t\,, \tag{28}$$

where $c > b > 0$.

The integral representation of ${}_2F_1$ is valid under the assumption that $|x| \leq 1$. In our case, it reduces to:

$$ {}_2F_1\left(\frac{1}{2}, m; m+1; x\right) = \frac{\Gamma(m+1)}{\Gamma(m)} \int_0^1 \frac{t^{m-1}}{\sqrt{(1-xt)}}\, \mathrm{d}t . \quad (29) $$

The average BER can then be given by

$$ P_e = \left(\frac{1}{2\sqrt{\pi}}\right) \frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m)} x^m \int_0^1 \frac{t^{m-1}}{\sqrt{(1-xt)}}\, \mathrm{d}t , \quad (30) $$

where

$$ x = \frac{2m}{2m+4\Omega}. \quad (31) $$

The substitution $(t = \sin 2(\theta)/x)$, is useful to solve the integral. Finally, we have for arbitrary value of m:

$$ P_e = \left(\frac{1}{\sqrt{\pi}}\right) \frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m)} \int_0^{\arcsin(\sqrt{x})} \sin^{2m-1}\theta ..\mathrm{d}\theta \quad (32) $$

and for integer m, Eq. (32) will result into:

$$ P_e = \left(\frac{1}{\sqrt{\pi}}\right) \frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m)} \sum_{n=0}^{m-1} \binom{m-1}{n} \frac{(-1)^n}{2n+1} \left[1 - y^{(n+\frac{1}{2})}\right], \quad (33) $$

where

$$ y = \sqrt{1-x} = \sqrt{\frac{4\Omega}{2m+4\Omega}}. \quad (34) $$

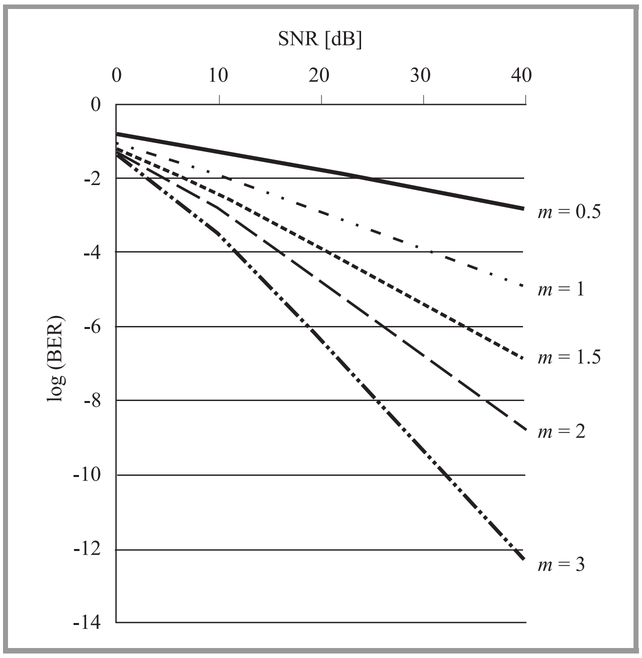

Fig. 7. The BER versus SNR of HPSK in Nakagami fading channels.

Special cases. The probability of error is computed for different value of m (Fig. 7). For the case of severe fading:

- $m = \frac{1}{2}$ (the half Gaussian distribution) Eq. (32) reduces to:
$$ P_e = \arcsin(\sqrt{x})/\pi , \quad (35) $$
- $m = 3/2$, Eq. (33) will reduce to:
$$ P_e = \left(\arcsin(\sqrt{x} - \sqrt{x(1-x)}\right)/\pi , \quad (36) $$
- $m = 1$ (the case of Rayleigh fading), the average probability of error in Eq. (33) reduces to:
$$ P_e = (1-y)/2 , \quad (37) $$
- $m = 2$, Eq. (33) will result to:
$$ P_e = (2 - 3y + y^3)/4 , \quad (38) $$
- $m = 3$ (very close to Rician fading), Eq. (33) will result to:
$$ P_e = \left(8 - (15y - 10y^3 + 3y^5)\right)/16 . \quad (39) $$

5. Conclusion

In this paper we come to an easy to evaluate expressions for the BER of HPSK performance in Nakagami fading channel as shown in the previous section. The cases of more severe fading than Rayleigh ($m = 1$) and Racian ($m = 3$) fading are considered. Figure 7 shows the probability of error of HPSK for different values of m. Also we proved that HPSK has SNR advantage of nearly 3 dB over QPSK.

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Mahmoud Youssuf is a Ph.D. candidate received his Master of science degree in electrical engineering at Alazhar University, Egypt. He is a Lecturer at the Department of Systems and Computers at Alazhar University. His research interests include issues related to wireless communications, federal networks, satellite communication applications. He is an author of research studies published at national and international journals, conference proceedings.

e-mail: mahmoud.youssuf@gmail.com
Alazhar University
Faculty of Engineering
Department of Systems and Computers
Republic Palace Square
Alawkaf Buildings A-1, 4th floor
Hadyek Elqubba, Cairo, Egypt

Mohamed Zaki Abdelmageed received his Ph.D. degree in electrical engineering. He is the chief of the Department of Systems and Computers at Alazhar University, Egypt. His research interests are related to networks, wireless communications and computer systems. He has published research papers at different national and international journals, conference proceedings.

e-mail: azhhar@mailer.eun.eg
Alazhar University
Faculty of Engineering
Department of Systems and Computers
Republic Palace Square
Alawkaf Buildings A-1, 4th floor
Hadyek Elqubba, Cairo, Egypt